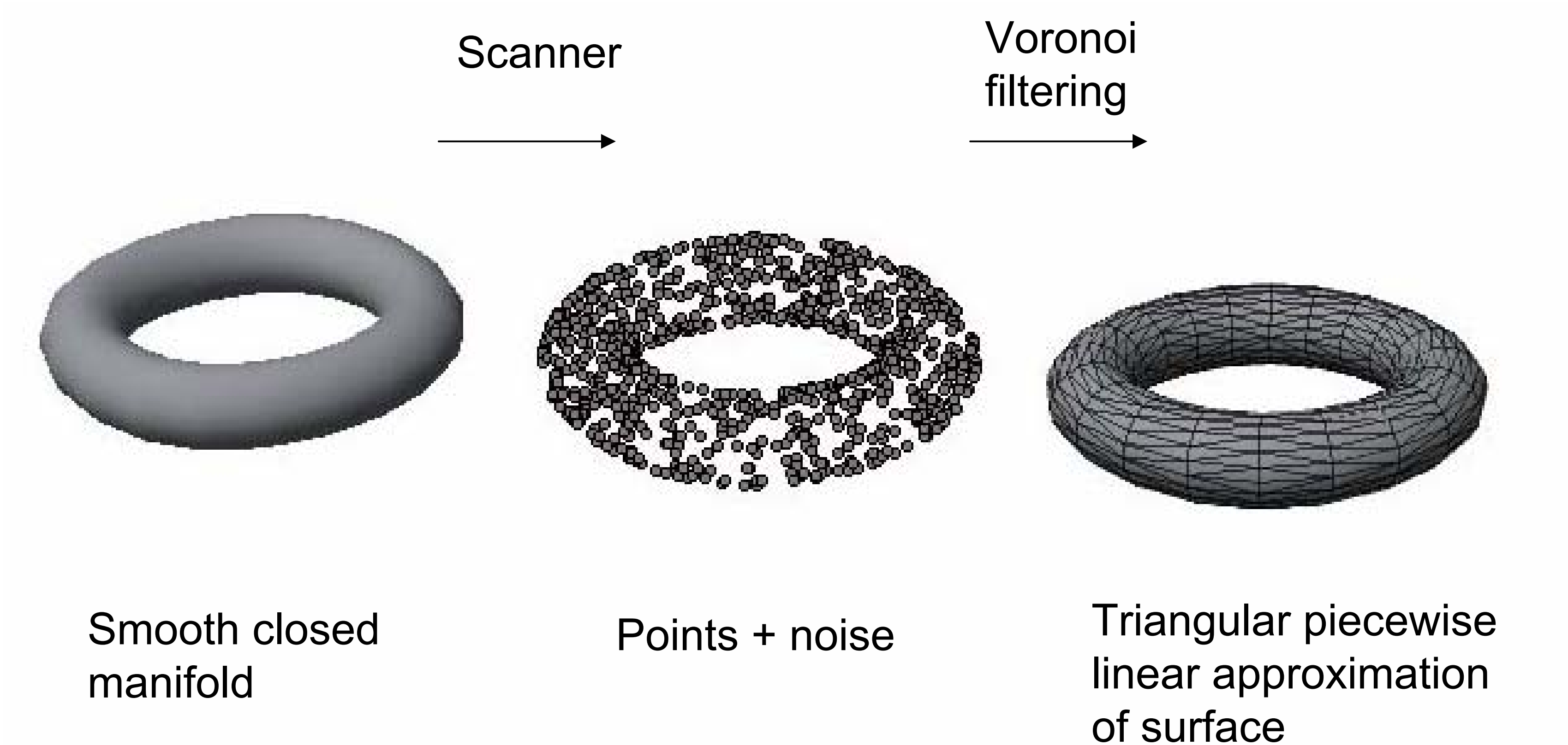
- A New Voronoi-Based Surface Reconstruction Algorithm

  Nina Amenta, Marshall Bern & Manolis Kamvysselis

- Surface Reconstruction by Voronoi Filtering

  Nina Amenta & Marshall Bern


Scanner
Voronoi filtering
Smooth closed manifold
Points + noise
Triangular piecewise linear approximation of surface

Key Points of Algorithm

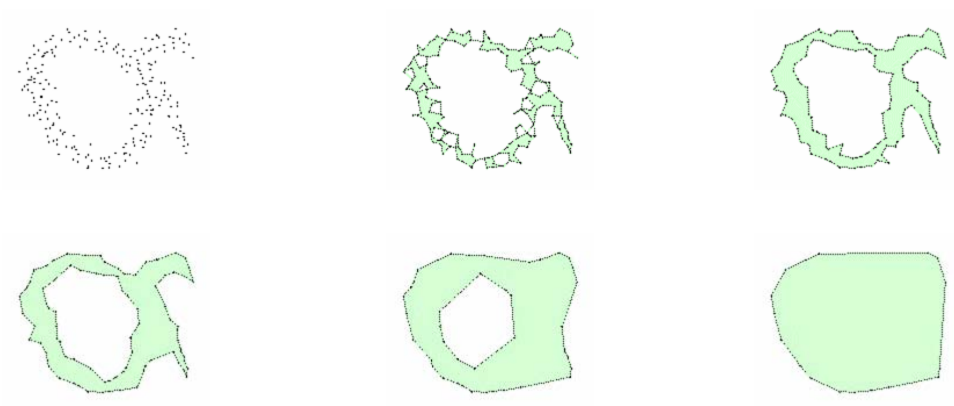
- Assumes smooth curve/surface without boundary
- Uses Delaunay triangulation
- Result exactly interpolates sample points
- Use medial axis to define "good" sample and to derive theoretical results
- If sample is "good", theory guarantees

  1) convergence to original surface in positions and normals
  2) result is topologically correct
- Allows nonuniform sampling

Related Work

α – Shapes (Edelsbrunner et al)

- Assigns a polyhedral shape to a point cloud S.
- Complement of the union of all spheres of radius α not containing S.

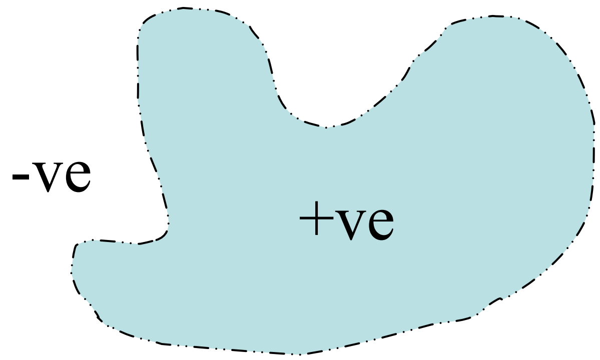


Comparison

- α must be chosen experimentally.
- No globally suitable value of α may exist.
- Voronoi filtering produces an inherently 2d “crust”.

Zero-Set Algorithms (Hoppe et al)

- Define a signed distance function on $\Re^3$
- Polygonalize zero-set using marching cubes
- Produces an approximating mesh



Comparison

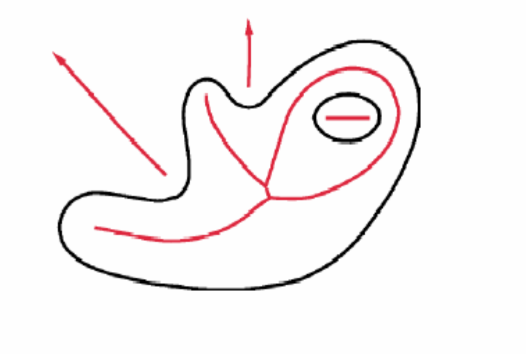
- Zero-set algorithm is faster, but crust is easier to implement
- Approximating rather than interpolating
- Low-pass filtering
- Sampling criterion and normal estimation ideas may be applicable to zero-set algorithms too

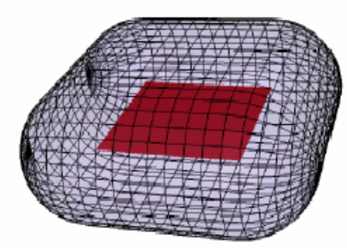
Medial Axis

Medial axis of a (d-1) dimensional surface F in $\Re^d$ is the set of points with more than one closest point in F.

In $\Re^2$ medial axis is a curve.

In $\Re 3$ medial axis is a surface.

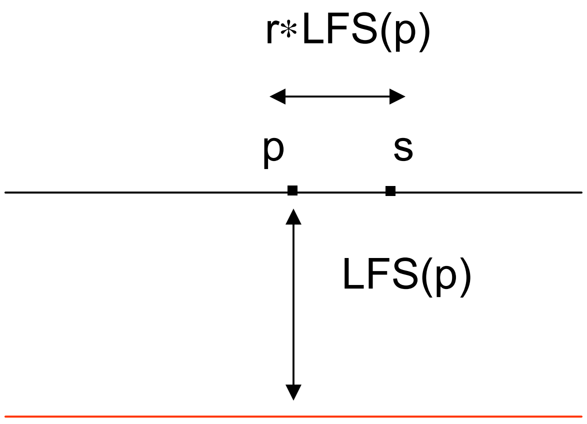




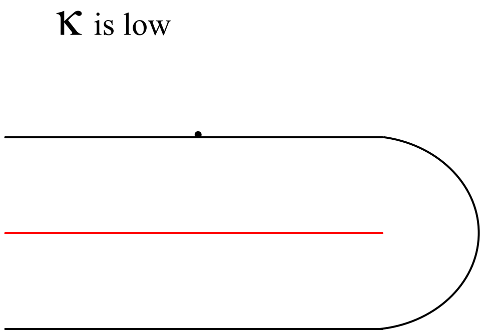
Local Feature Size of a point p on F (LFS(p)) is the minimum distance of p to the medial axis.

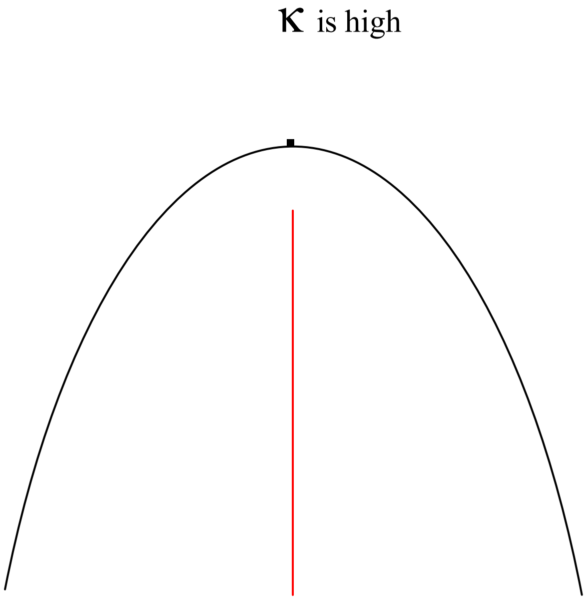
S is called an *r-sample* of F if every p in F has a sample within a distance $r*LFS(p), r<1$.

Intuitively, a sample is “good” if the sampling density is inversely proportional to the distance from the medial axis.



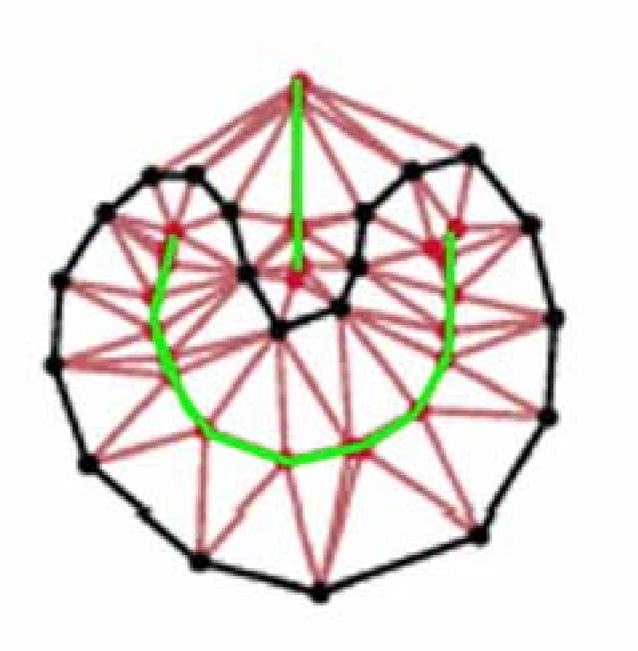
Using the medial axis accounts for both curvature plus proximity to other parts of the surface.





If medial axis touches the surface, theory states that we need infinitely dense sampling for convergence.

In 2D, the vertices of the Voronoi diagram of a dense sample approximate the medial axis.



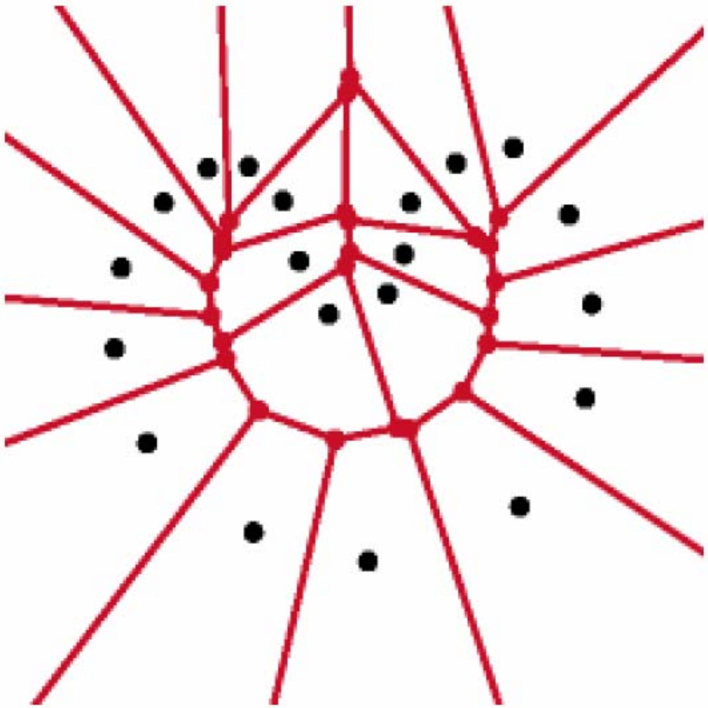
In 3D, this is not true. Need to use the “poles” of the Voronoi cells.

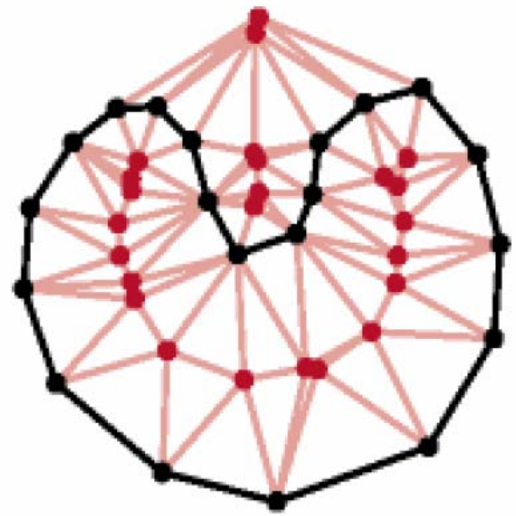
Algorithm in 2D

Input - Set of sample points S in $\Re^2$.

Output - Set of edges connecting points in S.

1. Compute Voronoi vertices V of S
2. Calculate Delaunay of $V \cup S$
3. Pick edges (p,q) where both p,q in S -- "Crust"
   Voronoi "filtering": adding Voronoi vertices filters out unwanted edges





2D Theoretical Results

Theorem 1:

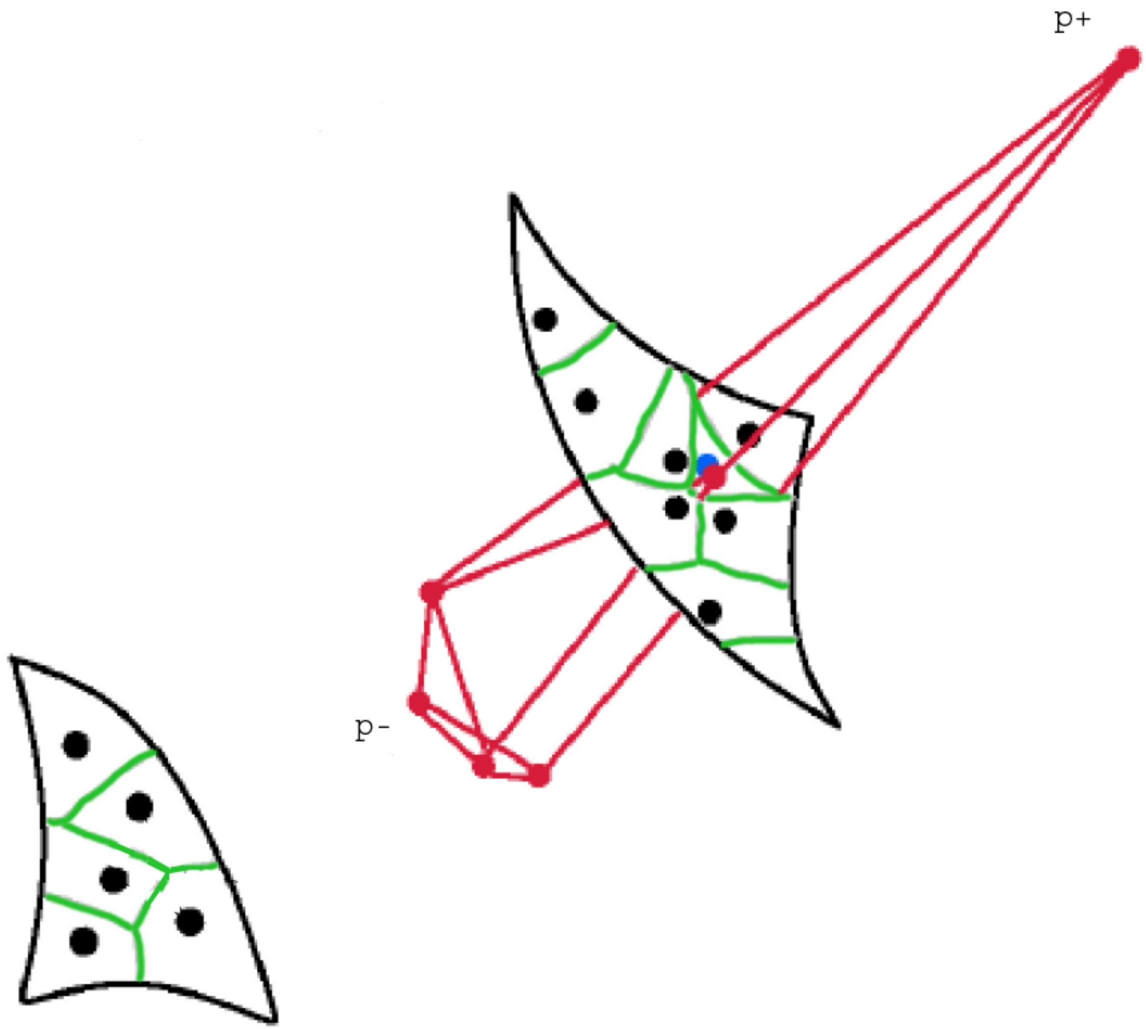
Given the crust of an r-sample S from a smooth curve F,

$r \leq .40$ => crust includes all edges between pairs of sample points
$r \leq .25$ => exactly

Theoretical guarantee that we have an accurate polyline reconstruction of F if the sample set fulfills this criteria.

Extension to 3D

- Not straight forward 2D extension. 5 theorems.
- Voronoi cell vertices don’t necessarily lie near medial axis
  - use 2 Voronoi poles not all Voronoi vertices
- Need two post processing steps on “raw crust”
  - Normal filtering
  - Manifold extraction


p+
p-

Theoretical *results in 3D*

Theorem 2:

If S is an r-sample of F for $r \leq .1$, then good triangles form a polyhedron homeomorphic to F.

Theorem 3:

If S is an r-sample of F for $r \leq .1$, the crust includes all good triangles.

Theorem 4:

If S is an r-sample of F for $r \leq .06$, the crust lies within a fattened surface by placing a ball of radius $5r*LFS(q)$ around each q in F.

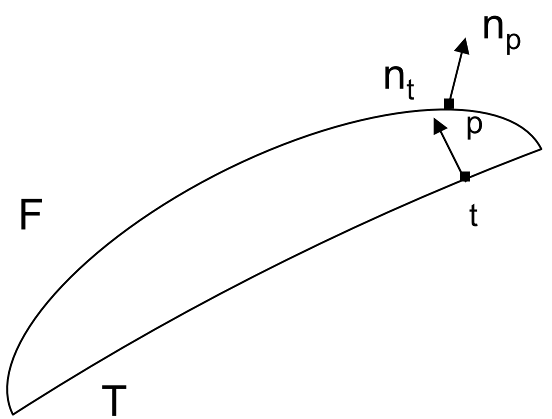
Guarantees that the raw crust contains a triangular mesh (the good triangles) that is topologically equivalent to F.

The crust can be brought arbitrarily close to the surface as r gets smaller.

Does not guarantee raw crust is a manifold or converges in surface normals.

Theorem 5:

Assume S is an r-sample and set θ = 4r. Let T be a triangle of the θ crust, where the θ crust is obtained after normal trimming and let t be any position on T. The angle between the normal of T and the normal of F at the point p in F closest to t = $O(\sqrt{r})$ radians.



acos (np·nt)= $O(\sqrt{r})$

Guarantees that after the normal filtering phase, the trimmed converges in the surface normal.

Theorem 6:

Assume S is an r-sample and set $\theta = 4r$. After the θ crust has been trimmed by the manifold extraction step, the trimmed crust is homeomorphic to F.

Guarantees we have a manifold that is topologically equivalent to the surface.

Raw Crust

- Not all Voronoi vertices are close to medial axis.
- Only use "poles" -- 2 Voronoi vertices farthest from s on opposite ends of F.

  p+ - Farthest vertex of cell from s

  p- - Farthest vertex of cell such that $sp^- \cdot sp^+ < 0$
- Compute Voronoi diagram of S.
- Find all poles P.
- Compute Delaunay triangulation of $S \cup P$.
- Extract triangles for which all 3 vertices are in S to get the "raw" crust.

- So far we have only guaranteed pointwise convergence.
- Triangle normals may be off (small triangles normal to the surface).
- Surface may not be a manifold (flat tetrahedron).
- Two more steps required: Normal Filtering and Manifold Extraction.

Normal Filtering

- Lemma states that $n^+ = sp^+$ and $n^- = sp^-$ are nearly orthogonal to the surface at S.

- Throw away triangles whose normals are too different from n^+ or n^-.

- For each vertex in a triangle, inspect the angle between the triangle normal and the vector to p^+ and the vertex.

- Throw away triangle if largest angle vertex > θ
  other vertex angles > 3θ/2

- θ is connected to r. Since r is unknown, requires experimentation.

- Normal filtering is dangerous around boundaries & sharp edges since n^+ and n^- are not normal to all tangent planes and good triangles can be filtered out.

- Perhaps n^+,n^- can be used as an estimate of tangent plane for zero-set methods.

Manifold Extraction (Trimming)

- Remaining triangles are roughly parallel to the surface but some may be oriented incorrectly. For example, a sliver.

- Two steps:
  - Orient all triangles consistently
  - Remove triangles on sharp edges

- Triangle orientation
  - Pick an s on convexhull (S)
  - Call p^+ outside, p^- inside
  - Pick a triangle incident to s and orient its pole to agree
  - Continue via breadth first search

- *Sharp edges*
  - A sharp edge is an edge where two successive incident triangles have a dihedral angle $>3\pi/2$. An edge with only 1 triangle is considered sharp.
  - Remove all triangles with sharp edges.

Algorithm Summary

Input: Sample point S.

1. Compute Voronoi diagram of S
2. For each sample point s and its Voronoi cell V_s
   a) If s not on convexhull p^+ = vertex of V_s farthest from s
   $n^+ = sp^+$
   b) If s on convexhull n^+ = average of outer normals of triangles
   c) let p^- be the Voronoi vertex of V_s that is farthest from s satisfying $sp^- \cdot n^+ < 0$
3. Compute Delaunay triangulation of $S \cup \{p^+, p^-\}$
4. Voronoi Filtering: Keep triangles whose vertices are in S
5. Normal Filtering
6. Trimming

Complexity

- $O(n^2)$, $n=|S|$

  Compute Voronoi Diagram of S

  Compute 3D Delaunay triangulation of $S \cup \{p+, p-\}$

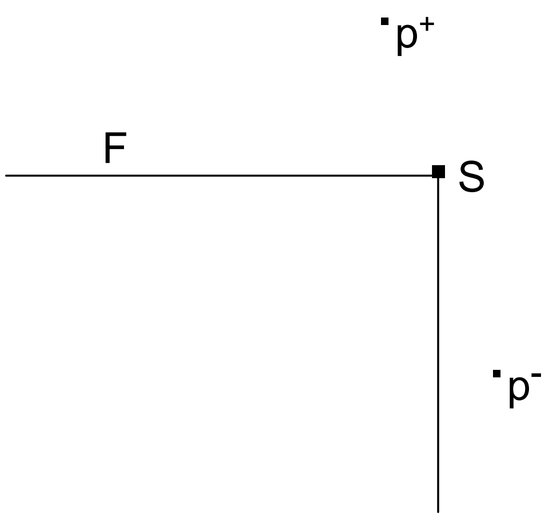
- Everything else requires linear time.

Numerical Issues

- Use exact Delaunay triangulation (Hull).
- Compute Voronoi vertices from Delaunay tetrahedra by solving a 4x4 system.
- If matrix condition number is poor, assume tetrahedra is flat and reject the vertex.

Sharp Edges

- At sharp edges medial axis is close to the surface, theory predicts we need a high sample density.
- Assumption that Voronoi cells are long and thin is not correct.
- n^+ and n^- do not span all tangent planes.
- A pole might lie close to the surface than the medial axis and including it will punch a hole in the mesh.



Heuristic for sharp edges

- Choose farthest and second farthest Voronoi vertices irrespective of direction.
- Avoids gaps around new sharp edges but permits excess triangles in sharp corners.
- Still no guarantee of correct reconstruction.

Future Research

- Noise is a problem if it approaches sampling density. Perhaps a hybrid algorithm of Voronoi crusts and α shapes can construct a "thickened surface".

- Algorithm doesn't handle sharp edges & boundaries well.

- Incorporating surface normal data if available.

- Compression. Store model only using vertices and reconstruct connectivity.