

# Space

*Pat Hanrahan*

## On Being the Right Size

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**"The most obvious differences between different animals are differences of size, but for some reason zoologists have paid singularly little attention to them. In a large textbook of zoology before me I find no indication that the eagle is larger than the sparrow, or the hippopotamus bigger than the hare, though some grudging admissions are made in the case of the mouse and the whale. But yet it is easy to show that the hare could not be as large as a hippopotamus, or a whale as small as a herring. For every type of animal there is a most convenient size, and a large change in size inevitably carries with it a change of form"**

**J. B. S. Haldane**

## **On Being the in the Right Space**

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**“The most obvious differences between different visualizations are differences of space, but for some reason visualization scientists have paid singularly little attention to them. In a large textbook of visualization before me I find no indication that the log-log space is different than the log-linear space, or that the mercator projection is different than the azimuthal equidistant projection, though some grudging admissions are made in the case of the parallel and perspective projections. But yet it is easy to show that distances are difficult to estimate under perspective, or that data obeying a power law is easy to see in a log-log plot. For every type of visualization there is a most convenient space, and a change into the right space inevitably makes relationships clearer.”**

**P. Hanrahan**

## **Topics**

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**Cartographic projections and distortion**

**Graphs and lines**

**Phase spaces**

**Reorderable spaces**

# Cartographic Projections

## Lattitude-Longitude Projection

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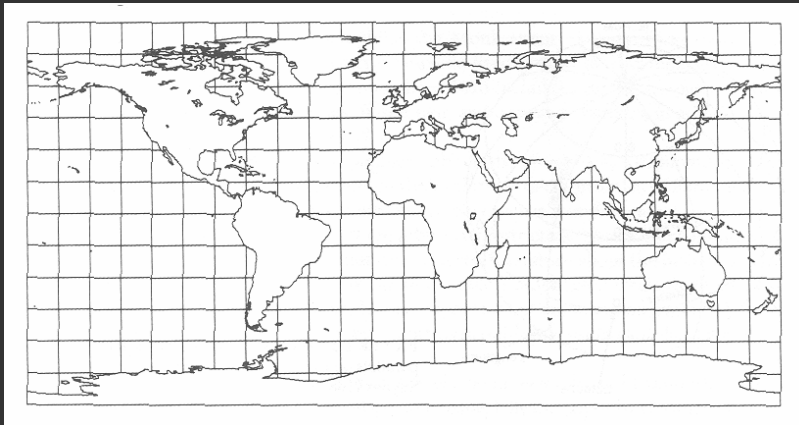


Figure 1.3, *Flattening the Earth*, Snyder

# Azimuthal Equidistance

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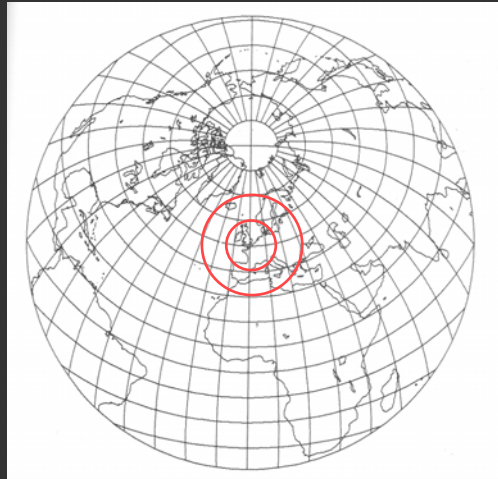


Figure 3.4, *Flattening the Earth*, Snyder

# Equi-Heading - Mercator

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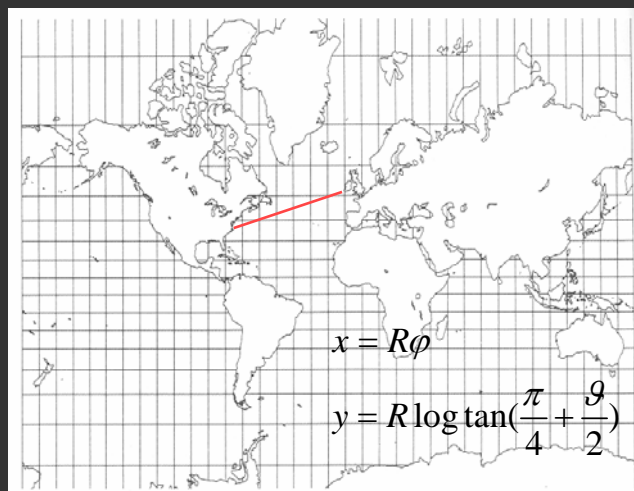


Figure 1.35, *Flattening the Earth*, Snyder

# Sinusoidal Equiareal Projection

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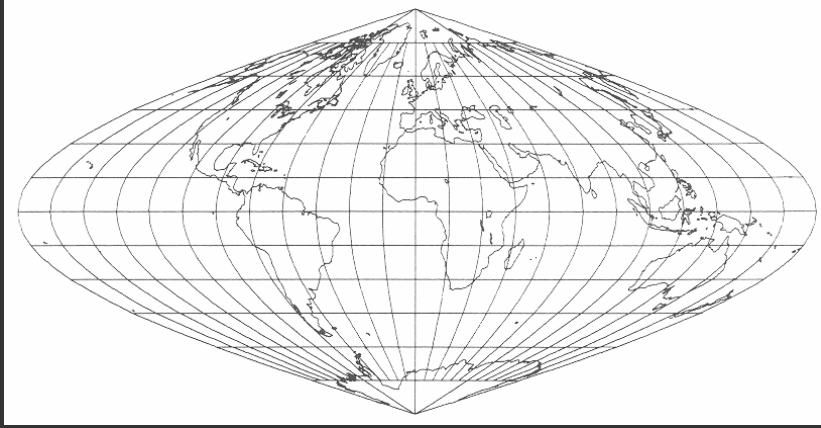
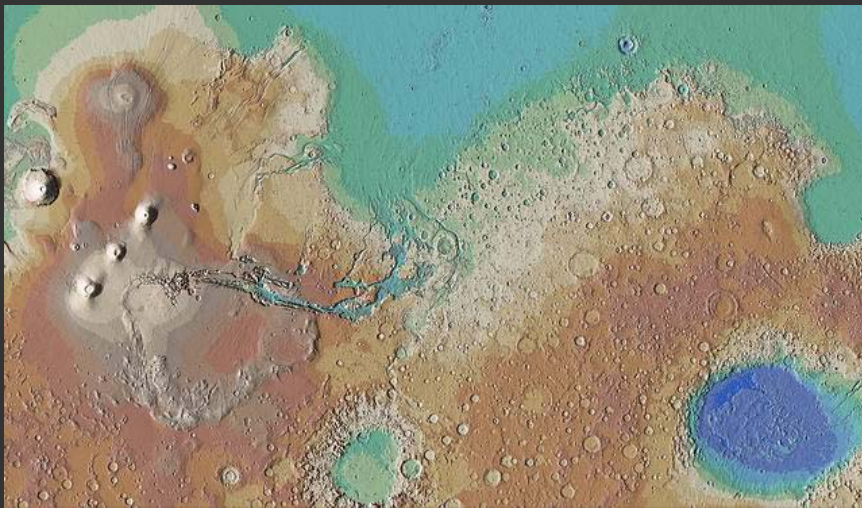


Figure 1.39a, *Flattening the Earth*, Snyder

# Mercator Projection of Mars

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Circular craters map to circles

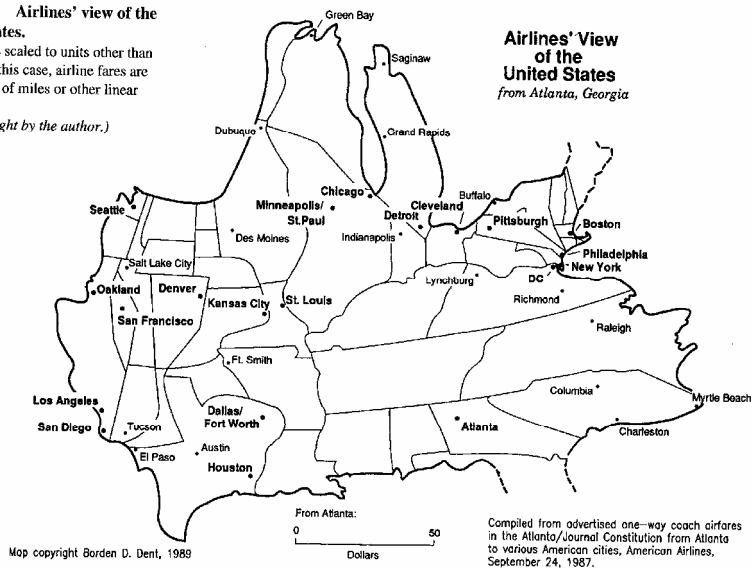


<http://astrogeology.usgs.gov/Gallery/MapsAndGlobes/mars.html#MarsMOLAContourMap>

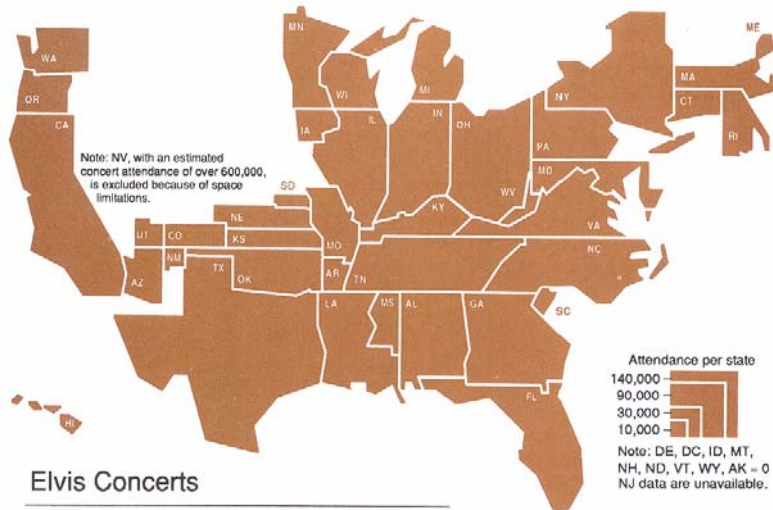
**Figure 1.8 Airlines' view of the United States.**

Maps can be scaled to units other than distance. In this case, airline fares are used instead of miles or other linear units.

(Map copyright by the author.)



**Scale Distance by Data**  
From *Cartography*, Dent



**Elvis Concerts**  
**Attendance per State, 1970 - 1977**

Source: Stanley, David E., with Frank Coffey. *The Elvis Encyclopedia*. Santa Monica, CA: General Publishing Group, Inc., 1994.

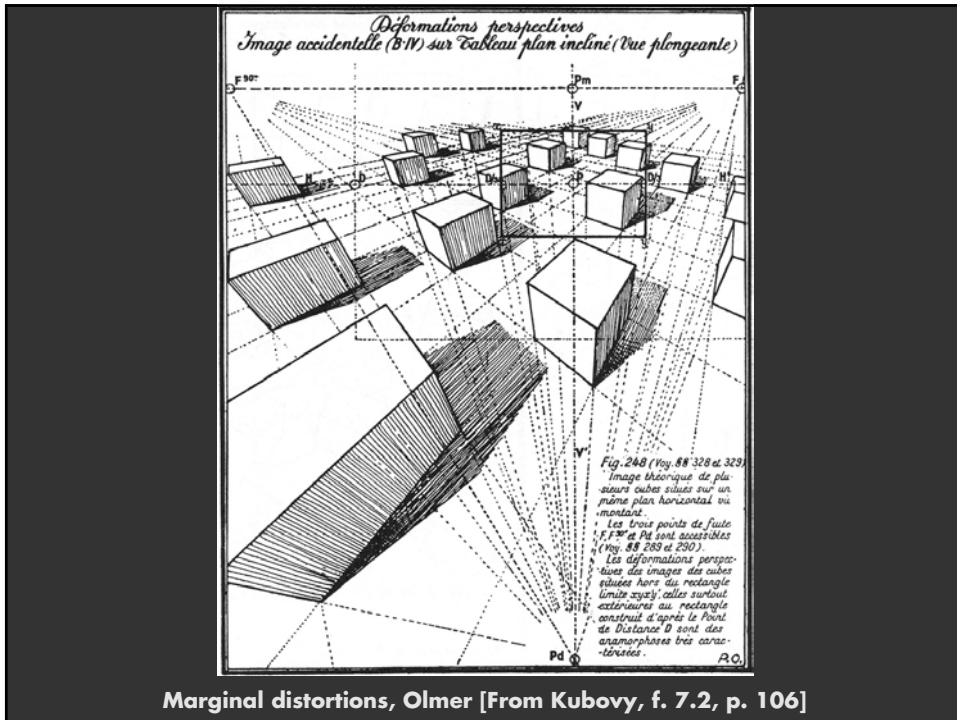
© 1995 Andrew Dent and Linda Turnbull

**Scale Area by Data**  
From *Cartography*, Dent



## Route Maps [Agrawala & Stolte]

1. Straighten wiggly lines
2. Snap turn directions to right angles
3. Expand regions with turns
4. Contract long straight roads
5. Label carefully to avoid clutter
6. Maintain overall orientation

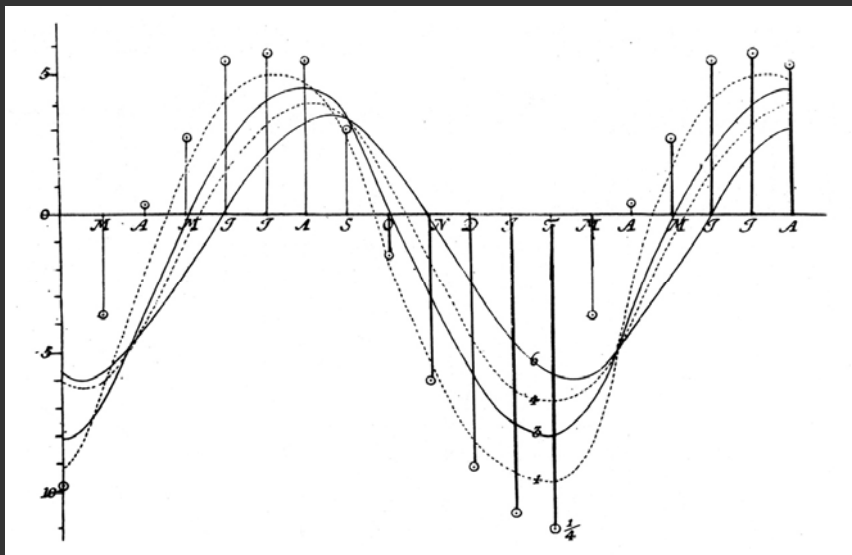


## Issues

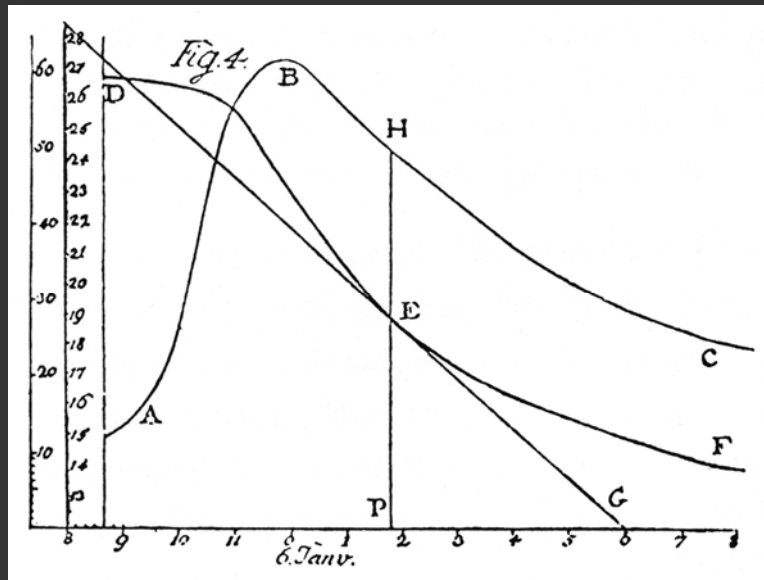
- Choose coordinate systems that support geometric reasoning
  - Anamorphosis: Maps features to lines
- Tension between geometric properties
  - Equiarea implies not equiangular
- People tolerate distortion -- to an extent
  - Maintain important information
  - Avoid extremes



# Graphs and Lines



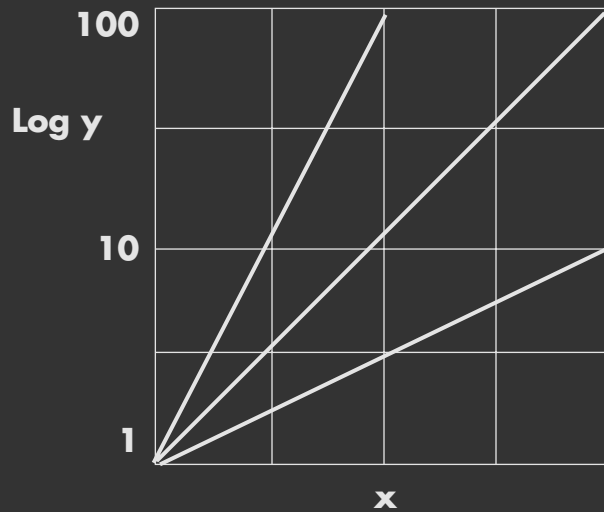
Johannes Lambert - 1765 [From Tilling]



Johannes Lambert - 1765 [From Tilling]

$$\log y = \log a + x \log b$$

Power Laws e.g. Stevens Power Laws



$$\log P = \log x + \log y$$

## From Batch to Interactive



## Anamorphosis

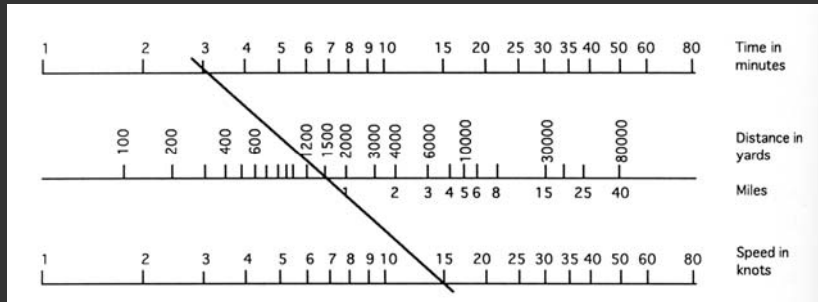
Tukey and Mosteller's pictures of power laws

Straightening out data

Best power law regression

# Nomograms

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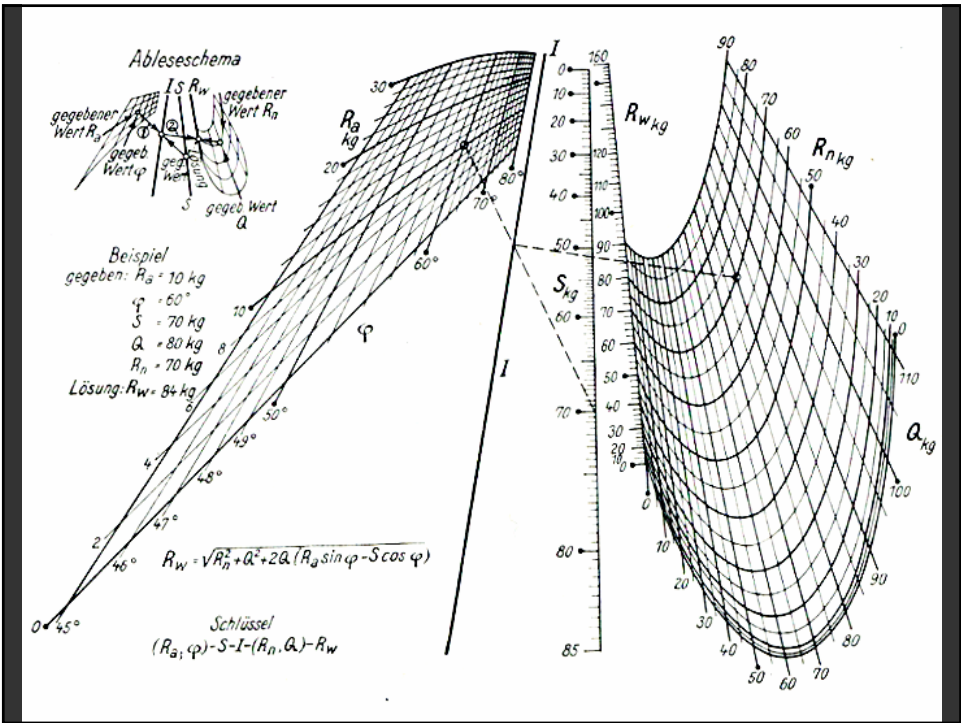
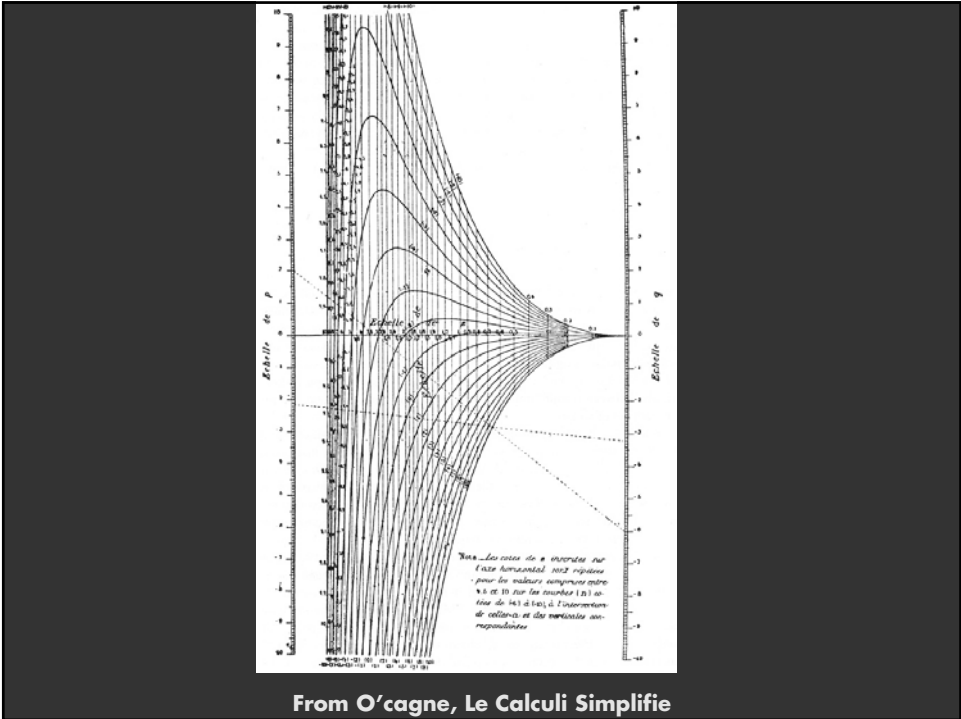


## The Rule of Three

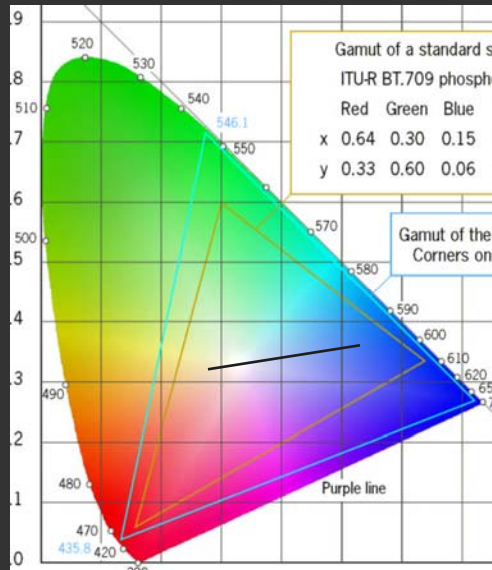
# Theory

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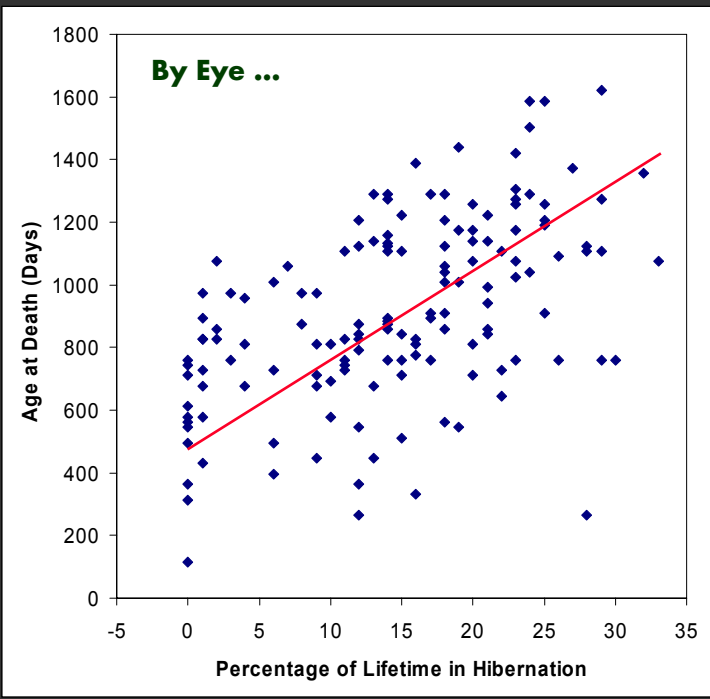
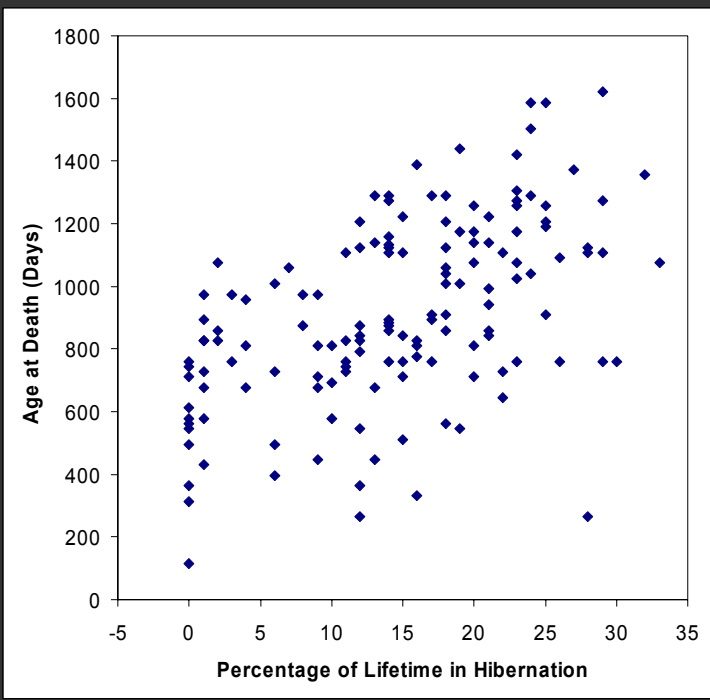
$$\begin{vmatrix} x_1(u) & y_1(u) & w_1(u) \\ x_2(v) & y_2(v) & w_2(v) \\ x_3(s,t) & y_3(s,t) & w_3(s,t) \end{vmatrix} = 0$$

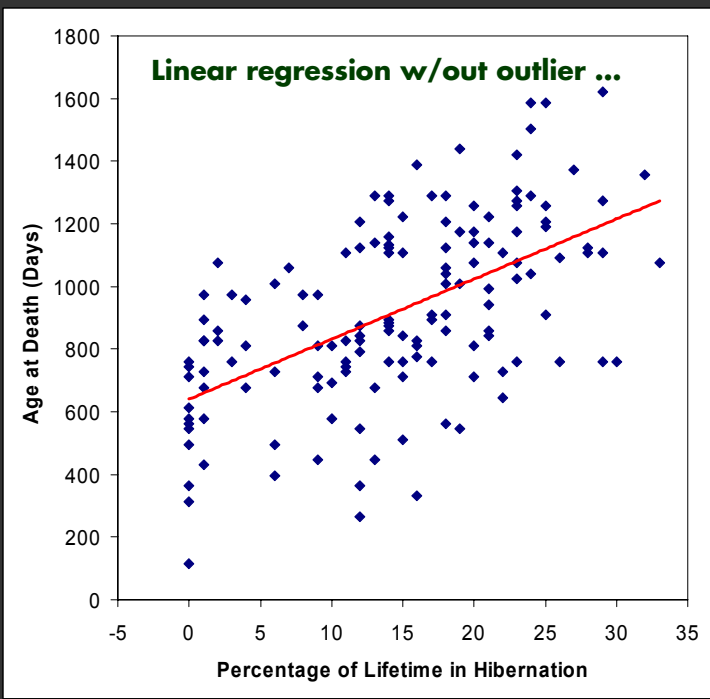
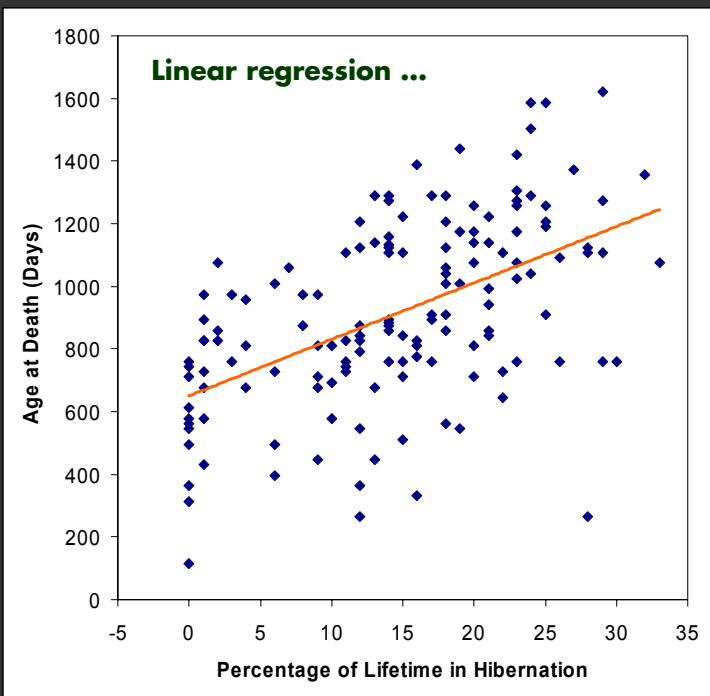


# 3D Lines Project to 2D lines



# Perception of Lines

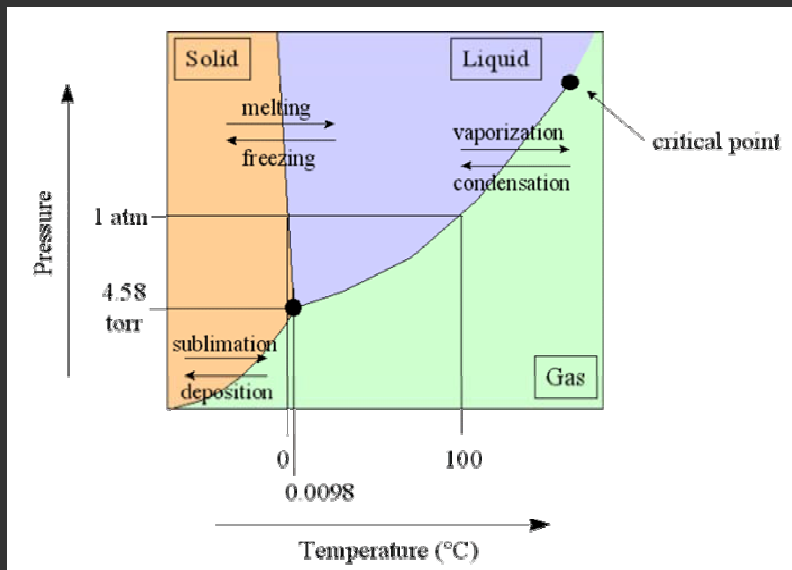






# Phase Spaces

## H<sub>2</sub>O Phase Diagram



# Cubic Filters

## Mitchell Cubic Filter

$$h(x) = \frac{1}{6} \begin{cases} (12 - 9B - 6C)x^3 + (18 - 12B - 6C)x^2 + (6 - 2B)x & |x| \leq 1 \\ (B - 6C)x^3 + (6B - 30C)x^2 + (12B - 48C)x + (8B - 24C) & 1 < |x| \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

**Properties:**

$\int_{-\infty}^{\infty} h(x) dx = 1$

B-spline: (1, 0)

Catmull-Rom: (0, 1/2)

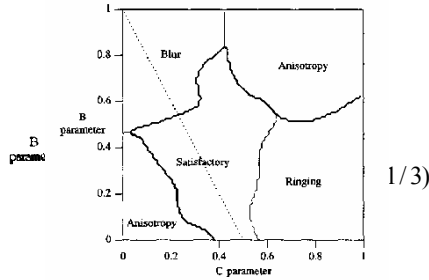


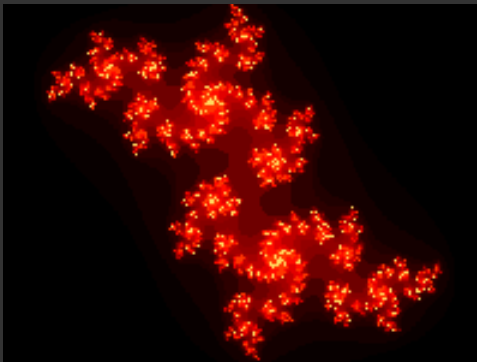
Figure 13. Regions of Dominant Subjective Behavior

From Mitchell and Netravali  
Look at other figures in that paper!

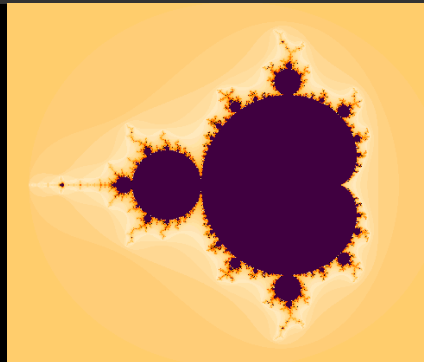
CS348B Lecture 7

Pat Hanrahan, Spring 2001

# Julia and Mandelbrot Sets



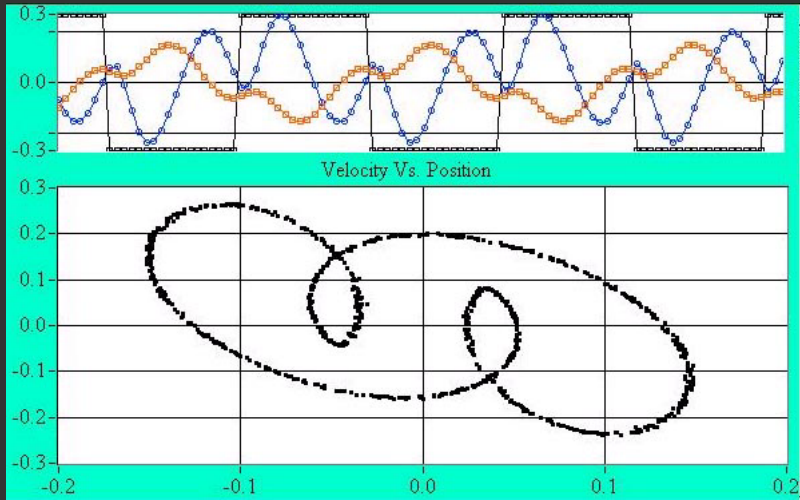
Julia Set



Mandelbrot Set

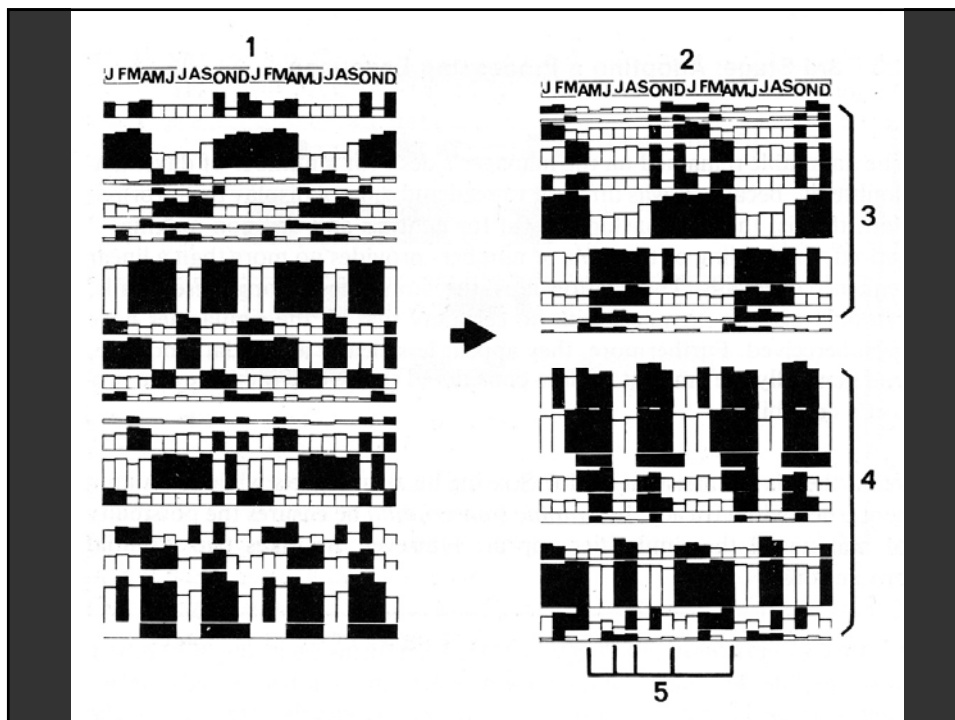
$$z^2 \leftarrow z^2 + c$$

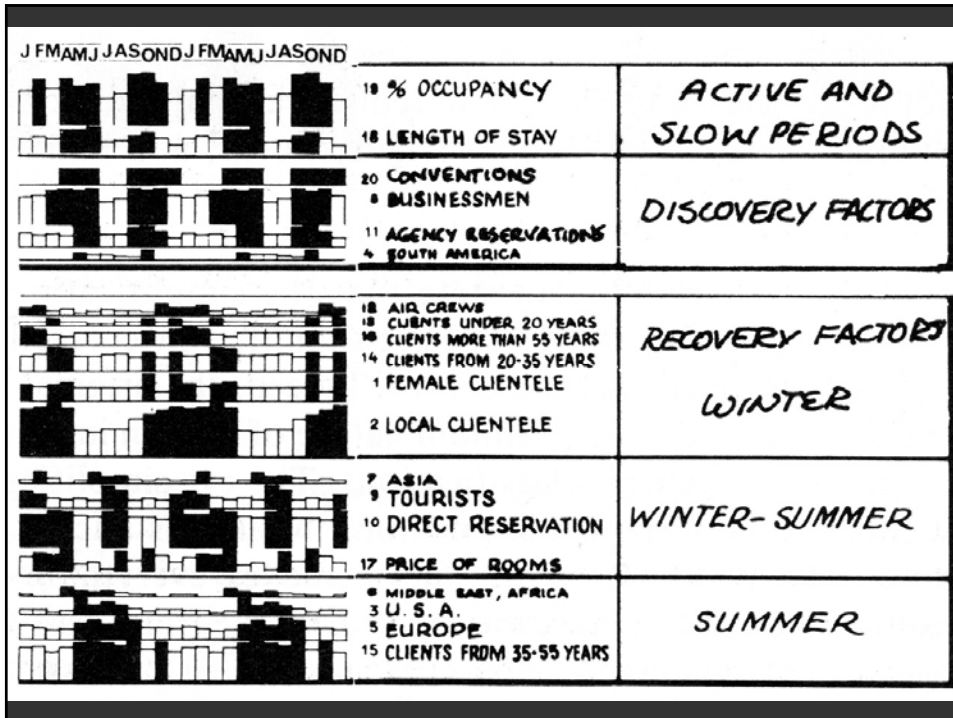
# Poincare Phase Space

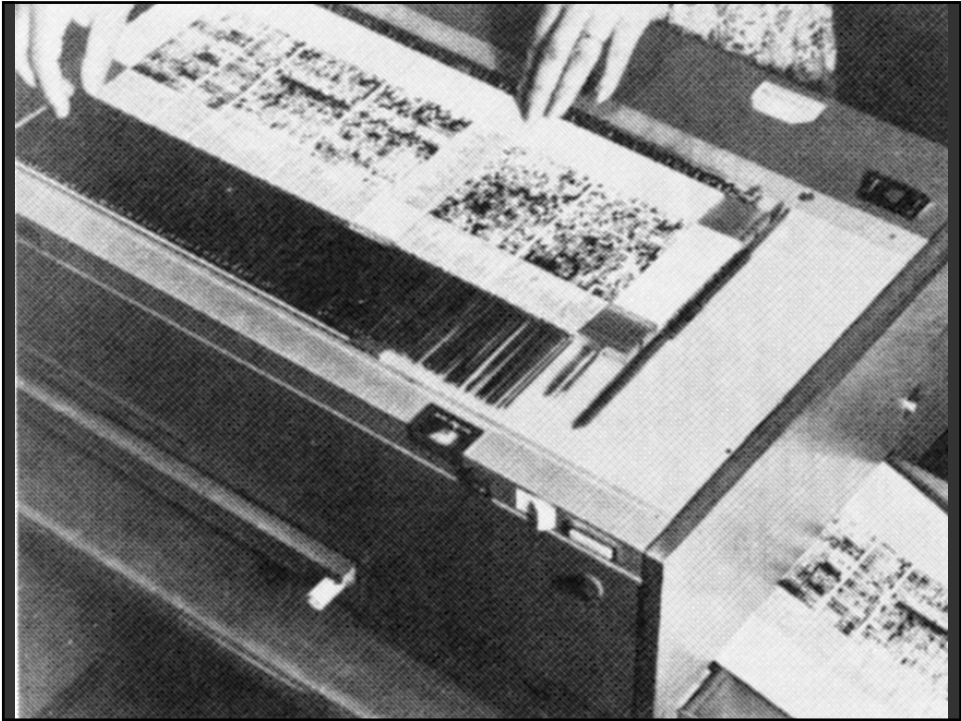


**Reorderable Spaces**  
**[From Bertin]**

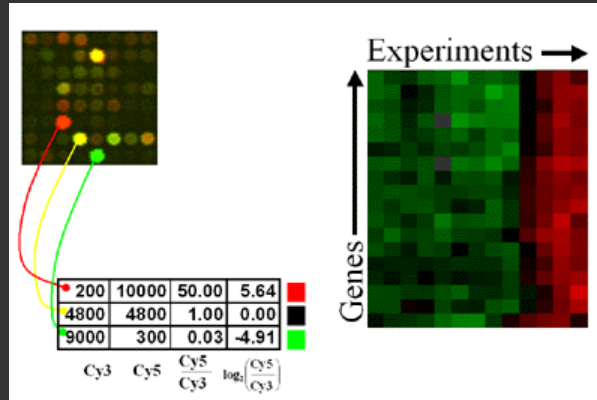
J	F	M	A	M	J	J	A	S	O	N	D		
26	21	26	28	20	20	20	20	40	15	40	1	% CLIENTELE FEMALE	
69	70	77	71	37	36	39	39	55	60	68	2	% —" — LOCAL	
7	6	3	6	23	14	19	14	9	6	8	3	% —" — U.S.A.	
0	0	0	0	8	6	6	4	2	12	0	4	% —" — SOUTH AMERICA	
20	15	14	15	23	27	22	30	27	19	19	5	% —" — EUROPE	
1	0	0	8	6	4	6	4	2	1	0	6	% —" — M.EAST, AFRICA	
3	10	6	0	3	13	8	9	5	2	5	7	% —" — ASIA	
78	80	85	86	85	87	70	76	87	85	87	8	% BUSINESSMEN	
22	20	15	14	15	13	30	24	13	15	13	9	% TOURISTS	
70	70	75	74	69	68	74	75	68	68	64	10	% DIRECT RESERVATIONS	
20	18	19	17	27	27	19	19	26	27	21	11	% AGENCY —" —	
10	12	6	9	4	5	7	6	6	5	15	12	% AIR CREWS	
2	2	4	2	2	1	1	2	2	4	2	13	% CLIENTS UNDER 20 YEARS	
25	27	37	35	25	25	27	28	24	30	24	14	% —" — 20-35 —" —	
48	49	42	48	54	55	53	51	55	46	55	15	% —" — 35-55 —" —	
25	22	17	15	19	19	19	19	20	19	22	16	% —" — MORE THAN 55 —" —	
163	167	166	174	152	155	145	170	157	174	165	17	PRICE OF ROOMS	
1.65	1.71	1.65	1.91	1.90	2.	1.54	1.60	1.73	1.82	1.66	18	LENGTH OF STAY	
67	82	70	83	74	77	56	62	90	92	78	19	% OCCUPANCY	
			X	X	X			X	X	X	20	CONVENTIONS	







# Clustering Gene Expression



## Nested Spaces

# Barley Data and the Trellis

		Glabron	Manchuria	No. 457	No. 462	No. 475	Peatland	Svansota	Trebi	Velvet	Wisconsin No.
Crockston	1931	38	40	46	49	44	42	40	47	41	50
	1932	26	33	34	31	32	25	21	42	32	36
Duluth	1931	30	29	34	28	33	32	26	34	26	32
	1932	26	23	23	23	27	31	22	31	22	29
Grand Rapids	1931	29	33	32	25	20	35	30	30	23	34
	1932	14	22	19	20	15	27	17	21	32	21
Morris	1931	29	27	29	30	23	30	26	44	26	29
	1932	35	34	44	47	44	43	35	47	39	47
University Farm	1931	43	27	43	37	25	33	35	37	40	39
	1932	37	27	26	26	30	28	27	29	27	38
Waseca	1931	55	49	58	66	47	49	47	64	50	59
	1932	38	33	42	45	41	36	39	49	37	58

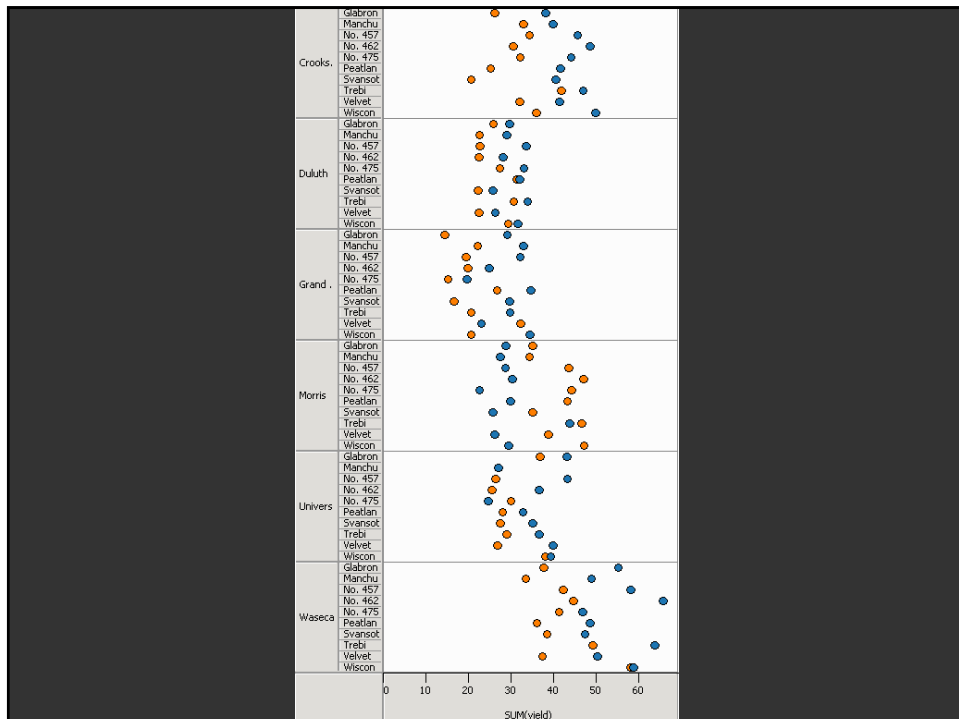
Yields per plot are measured

6 Sites = {Crockstein, Duluth, Grand Rapids, Morris, University Farm, Waseca}

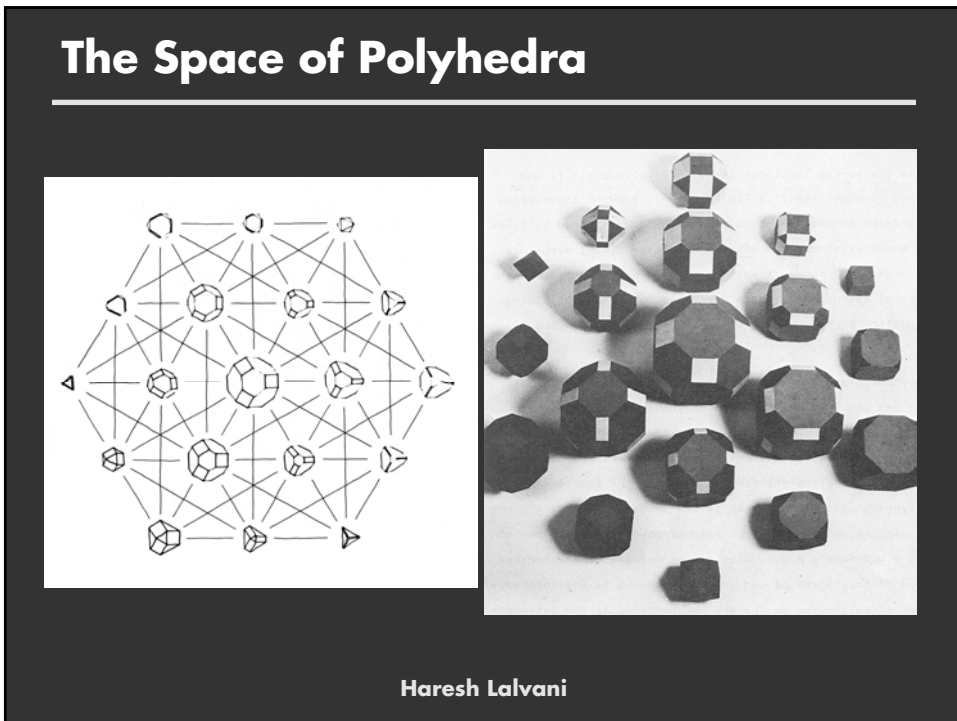
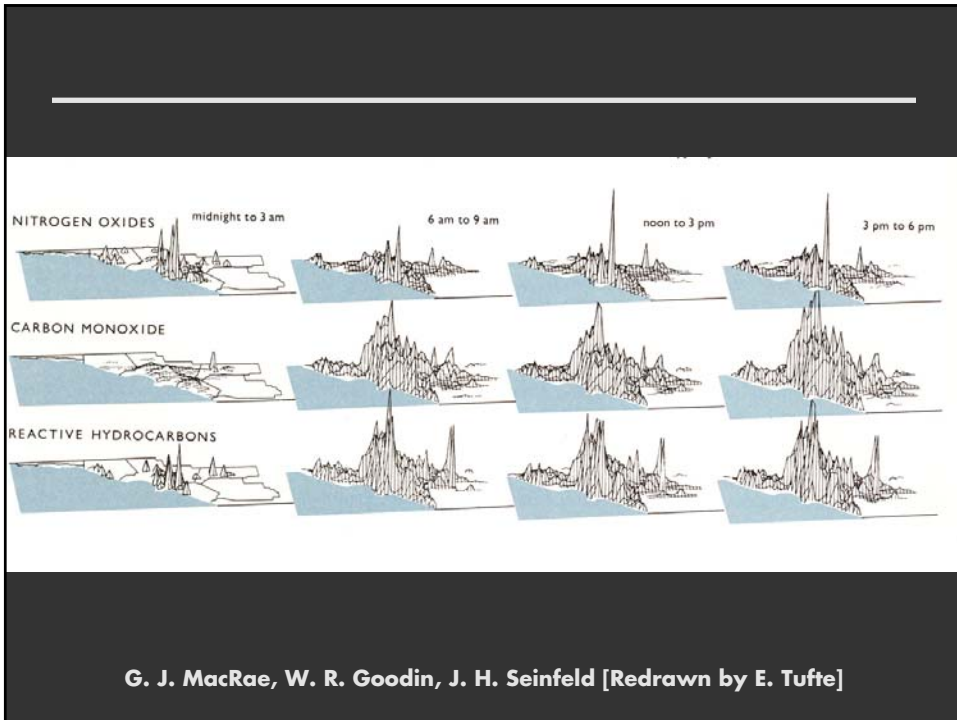
8 Varieties = {Glabron, Manchuria, No. 457, No. 462, No. 475, Peatland, Svansota, Trebi, Velvet, Wisconsin No. 38}

2 Years = {1931, 1932}

Example from Cleveland







# Wrap-Up

## Summary

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**On being in the right space**

**Spatial encoding the most important encoding**

**Geometric invariants of spatial transformations  
support geometric reasoning**

**“Linear” reasoning**

**The good and bad of distortion**

**Graphs and abstract spaces recent invention**