Limitations of lenses

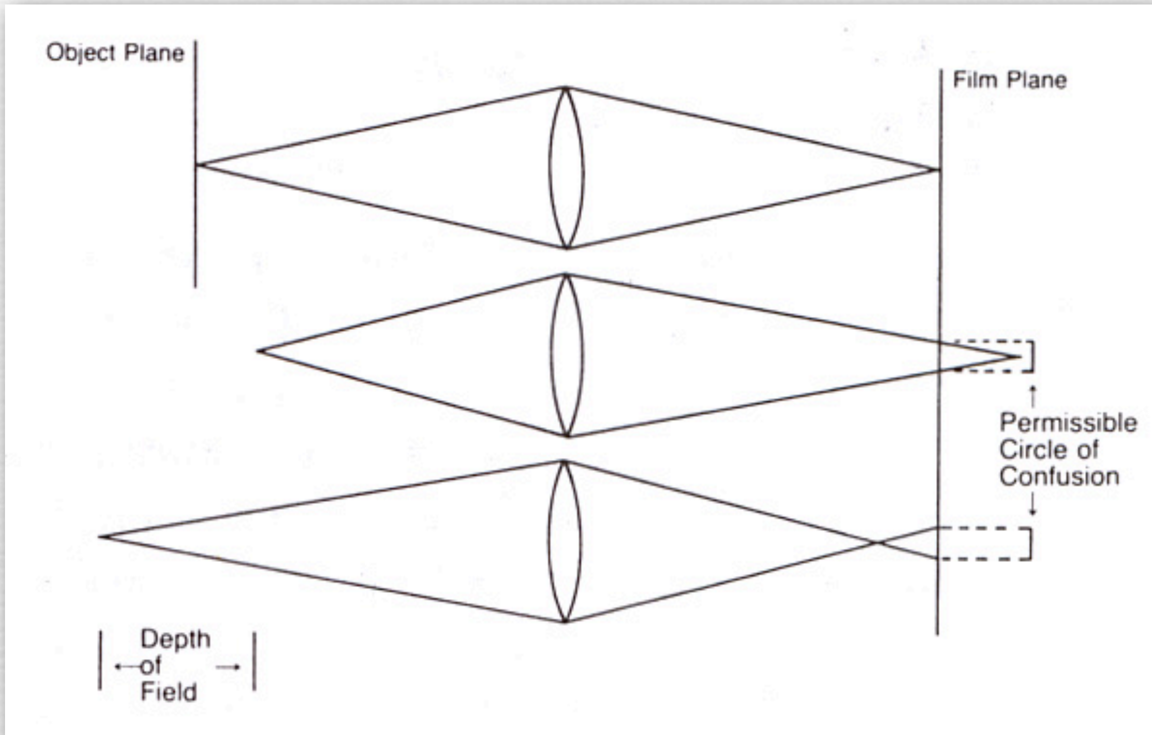
CS 448A, Winter 2010

Marc Levoy
Computer Science Department
Stanford University

Outline

- misfocus & depth of field
- aberrations & distortion
- veiling glare
- flare and ghost images
- vignetting
- diffraction

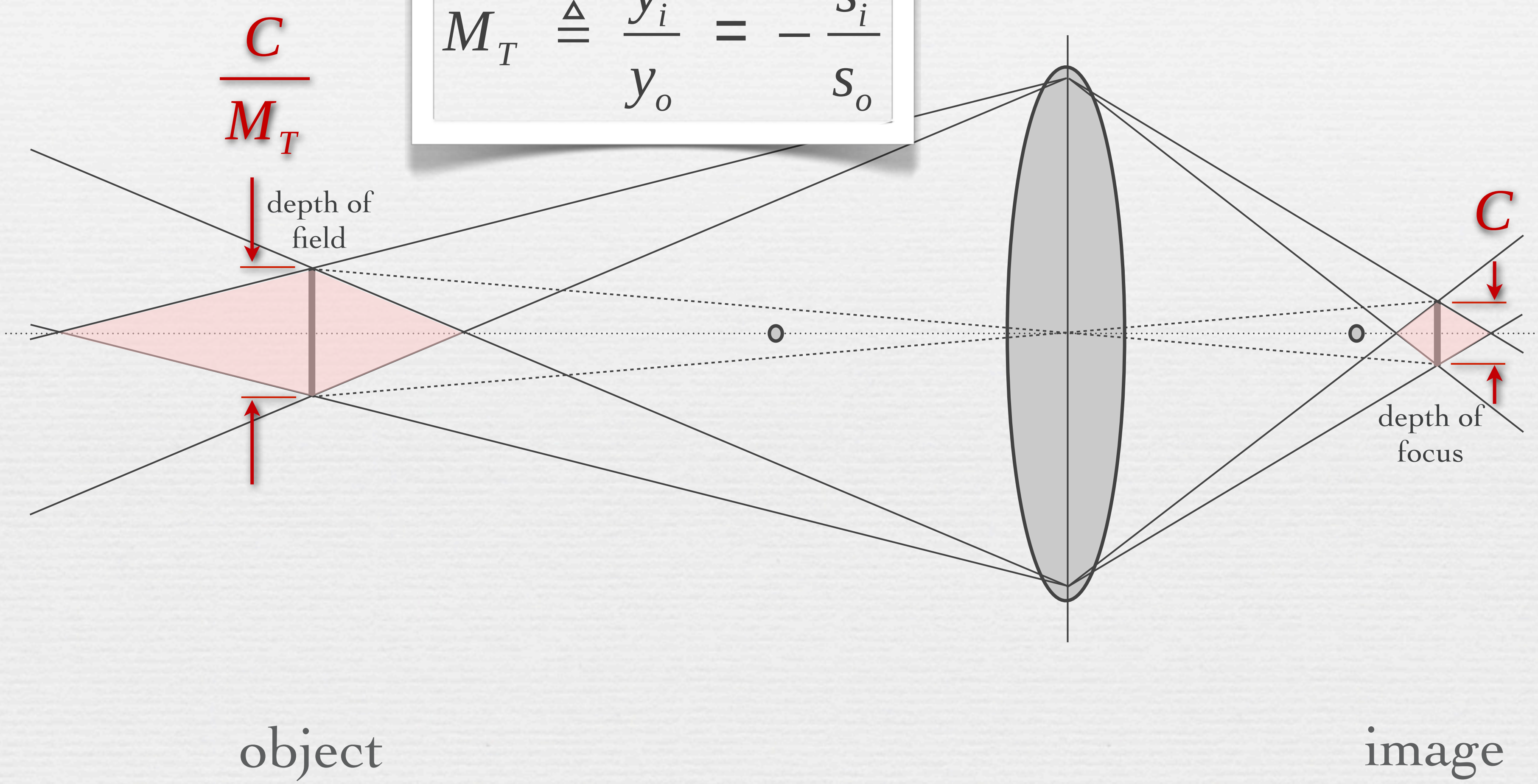
Circle of confusion (C)



- C depends on sensing medium, reproduction medium, viewing distance, human vision,...
  - for print from 35mm film, 0.02mm is typical
  - for high-end SLR, 6μ is typical (1 pixel)
  - larger if downsizing for web, or lens is poor

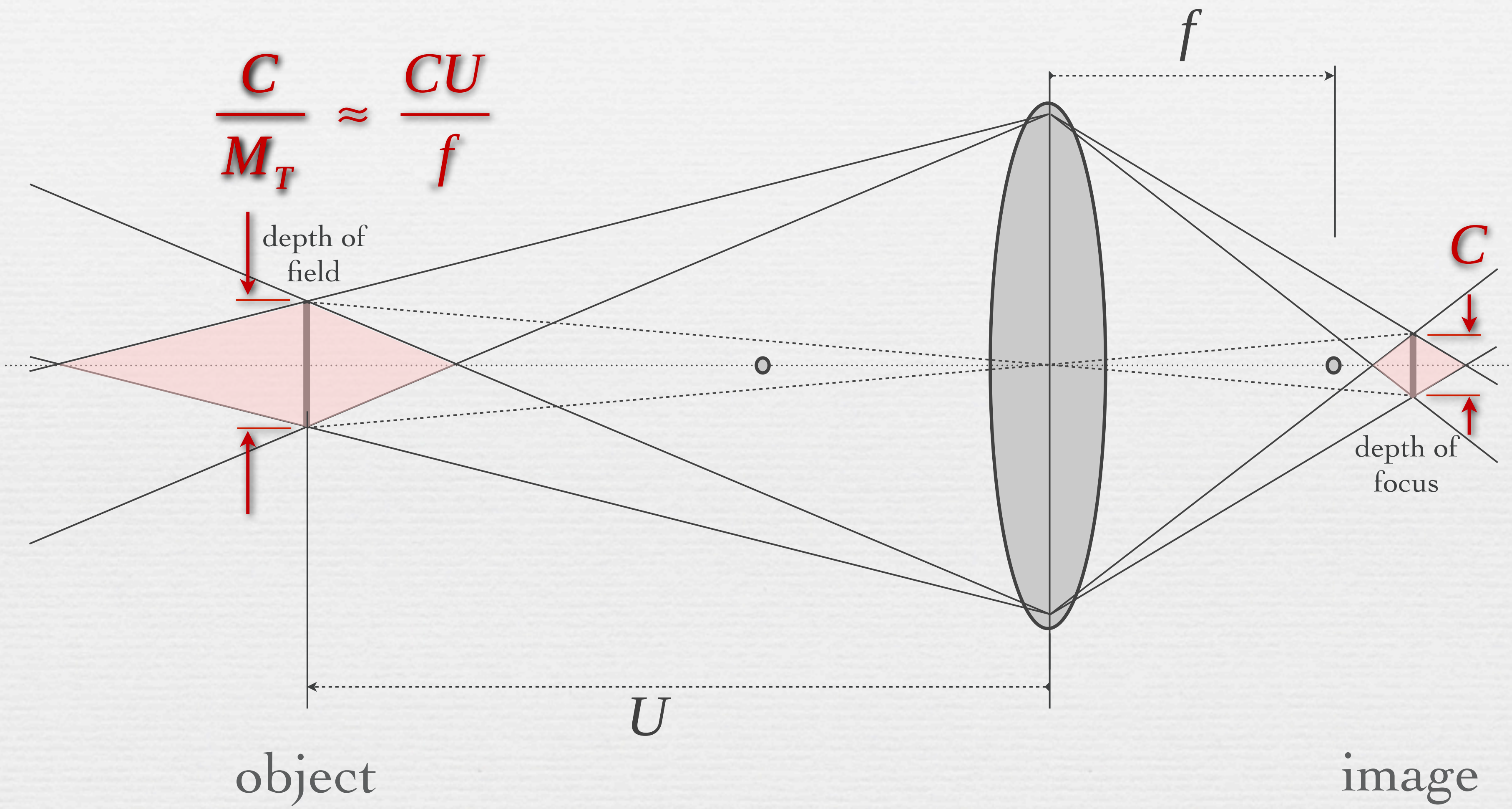
Depth of field formula

$$M_T \triangleq \frac{y_i}{y_o} = -\frac{s_i}{s_o}$$



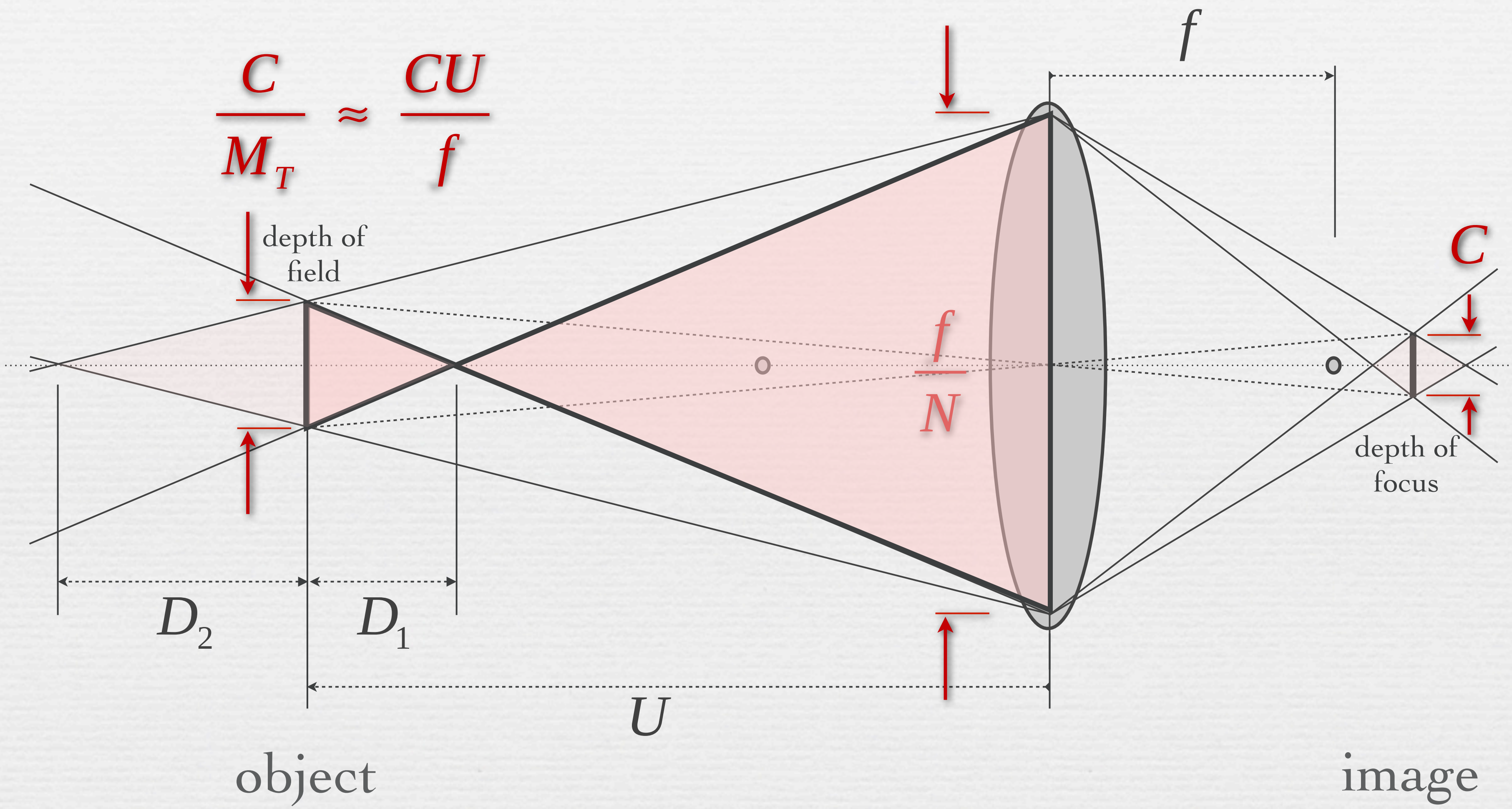
- DoF is asymmetrical around the in-focus object plane
- conjugate in object space is typically bigger than C

Depth of field formula



- DoF is asymmetrical around the in-focus object plane
- conjugate in object space is typically bigger than C

Depth of field formula



$$\frac{D_1 f}{CU} = \frac{U - D_1}{f/N} \quad \ldots\ldots \quad D_1 = \frac{NCU^2}{f^2 + NCU} \qquad D_2 = \frac{NCU^2}{f^2 - NCU}$$

Depth of field formula

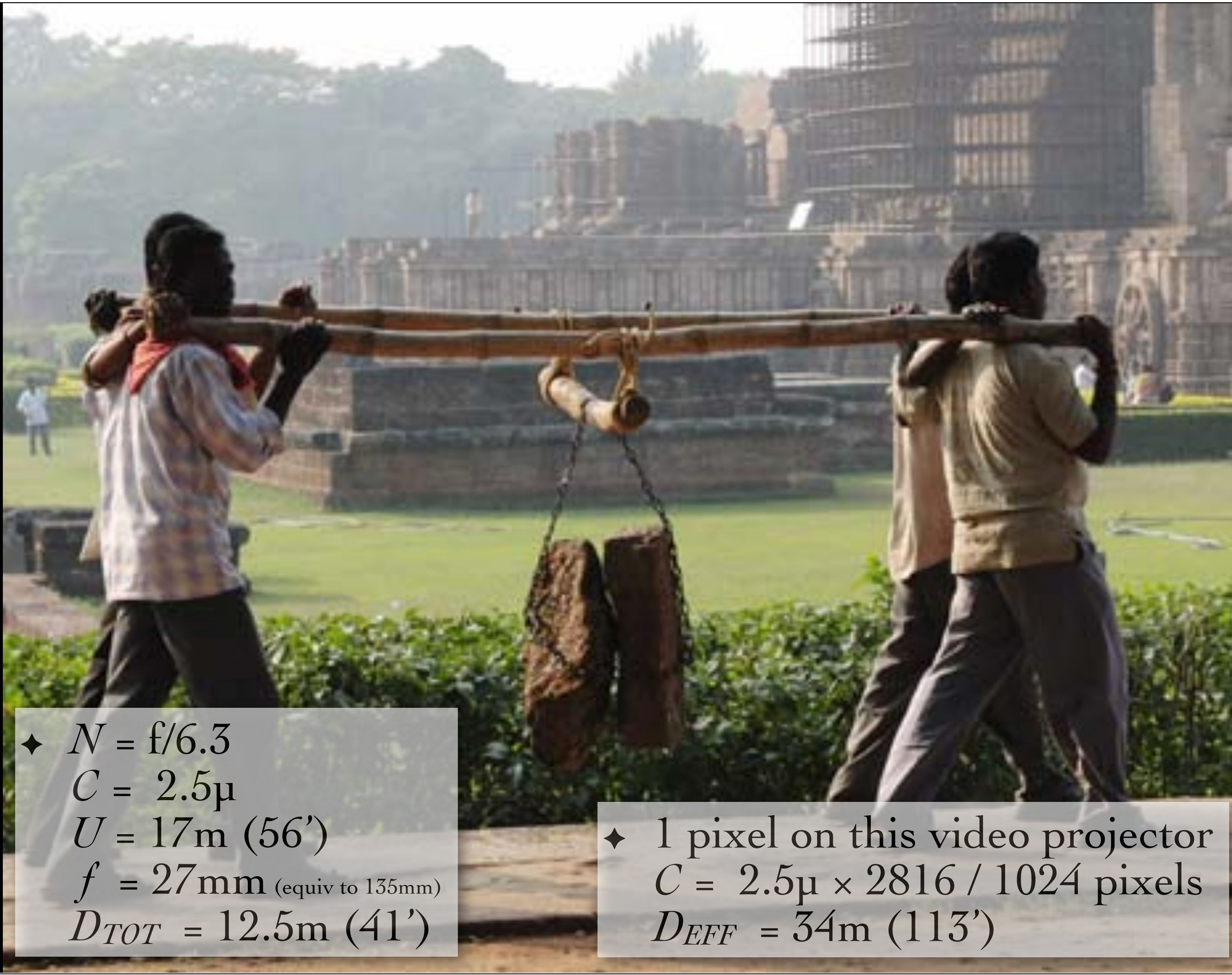
$$D_{TOT} = D_1 + D_2 = \frac{2NCU^2 f^2}{f^4 - N^2C^2U^2}$$

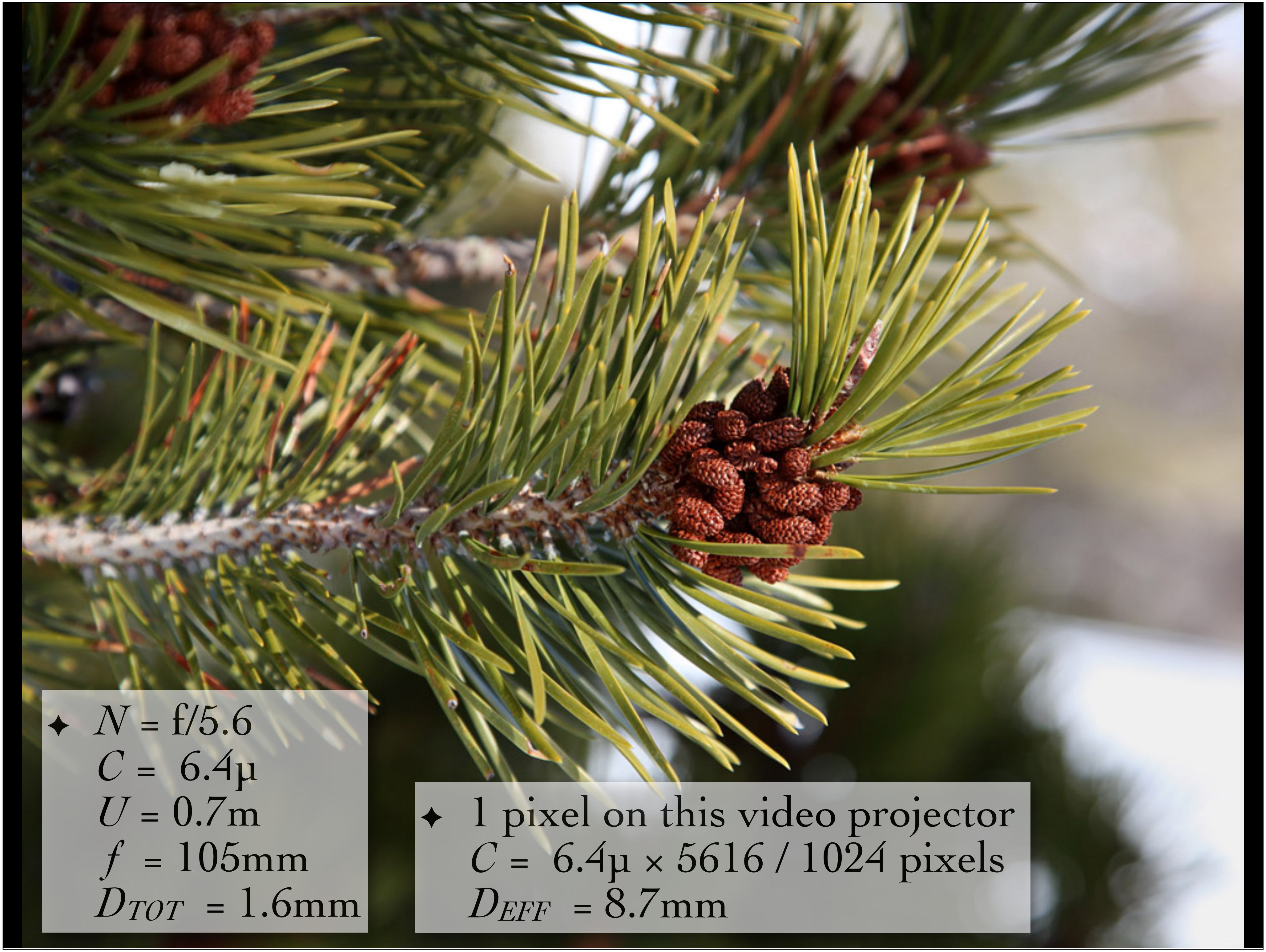
- $N^2C^2D^2$ can be ignored when conjugate of circle of confusion is small relative to the aperture

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

- where
  - N is F-number of lens
  - C is circle of confusion (on image)
  - U is distance to in-focus plane (in object space)
  - f is focal length of lens


$D_{TOT} \approx \frac{2NCU^2}{f^2}$
✦ N = f/4.1
C = 2.5µ
U = 5.9m (19’)
f = 73mm (equiv to 362mm)
D_{TOT} = 132mm
✦ 1 pixel on this video projector
C = 2.5µ × 2816 / 1024 pixels
D_{EFF} = 363mm


✦ N = f/6.3
C = 2.5μ
U = 17m (56’)
f = 27mm (equiv to 135mm)
DTOT = 12.5m (41’)
✦ 1 pixel on this video projector
C = 2.5μ × 2816 / 1024 pixels
DEFF = 34m (113’)



- N = f/5.6
  C = 6.4µ
  U = 0.7m
  f = 105mm
  D_{TOT} = 1.6mm

- 1 pixel on this video projector
  C = 6.4µ × 5616 / 1024 pixels
  D_{EFF} = 8.7mm

Canon MP-E
65mm 5:1 macro

(Mikhail Shlemov)

- N = f/2.8
  C = 6.4μ
  U = 31mm
  f = 65mm
  (use $N' = (1+M_T)N$ at short conjugates (M_T=5 here)) = f/16
  D_{TOT} = 0.048mm! (48μ)

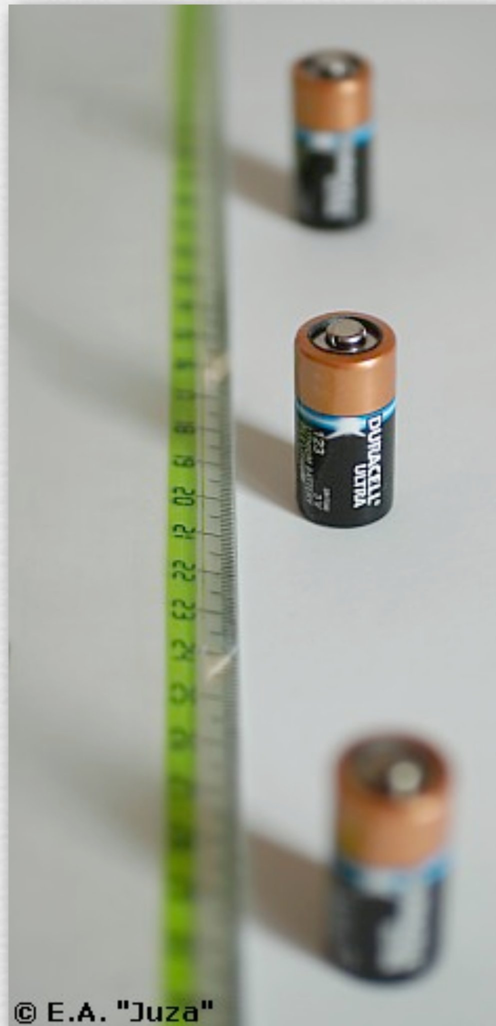
DoF is linear with aperture

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

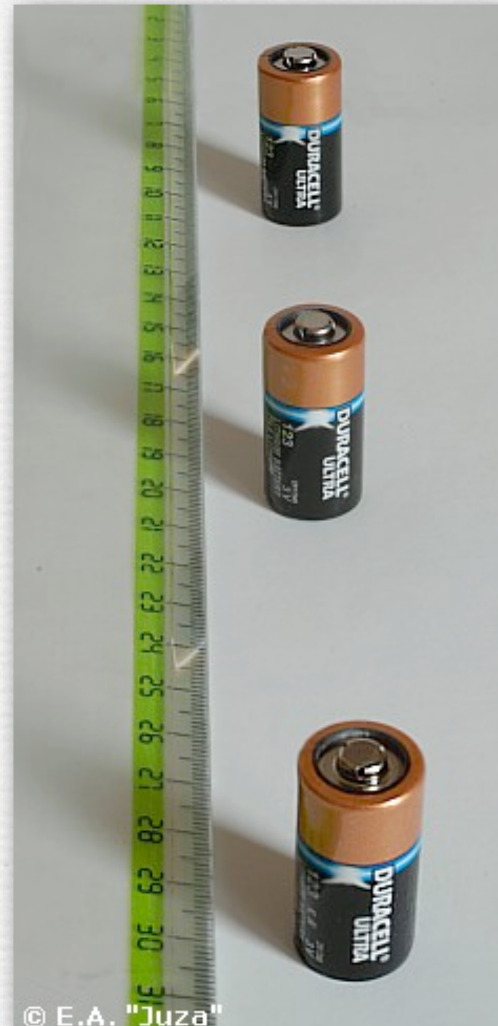
(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/dof.html

(juzaphoto.com)



f/2.8



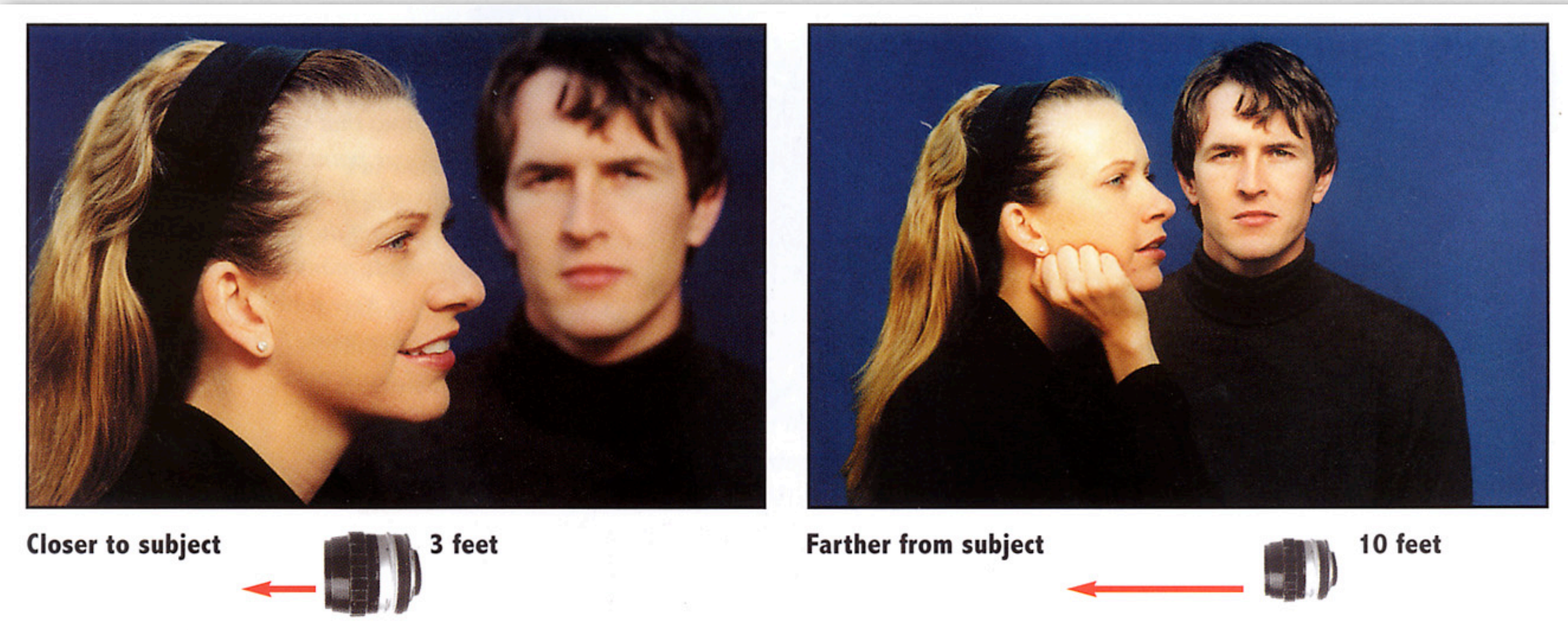
f/32

DoF is quadratic with focusing distance

$$D_{TOT} \approx \frac{2NC\boxed{U^2}}{f^2}$$

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/dof.html



(London)

Hyperfocal distance

- the back depth of field

$$D_2 = \frac{NCU^2}{f^2 - NCU}$$

- becomes infinite if

$$U \geq \frac{f^2}{NC} \triangleq H$$

- In that case, the front depth of field becomes

$$D_1 = \frac{H}{2}$$

- so if I had focused at 32m, everything from 16m to infinity would be in focus on a video projector, including the men at 17m

- N = f/6.3
  C = 2.5µ × 2816 / 1024 pixels
  U = 17m (56')
  f = 27mm (equiv to 135mm)
  D_{TOT} = 34m on video projector
  H = 32m (106')

(FLASH DEMO)

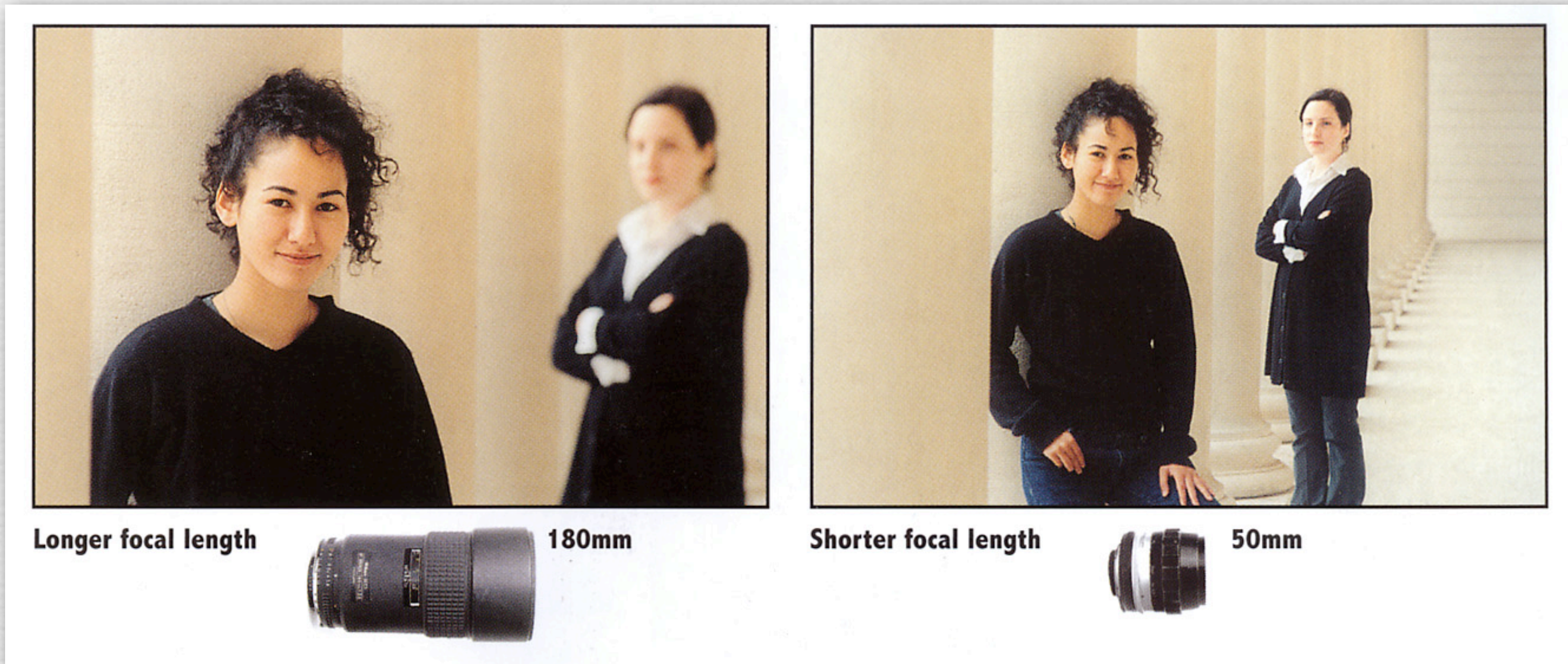
http://graphics.stanford.edu/courses/cs178/applets/dof.html

DoF is inverse quadratic with focal length

$$D_{TOT} \approx \frac{2NCU^2}{\boxed{f^2}}$$

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/dof.html



(London)

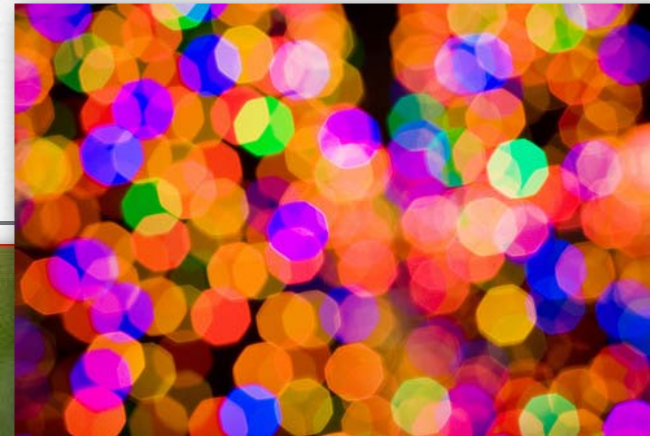
Q. Does sensor size affect DoF?

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

- as sensor shrinks, lens focal length f typically shrinks to maintain a comparable field of view
- as sensor shrinks, pixel size C typically shrinks to maintain a comparable number of pixels in the image
- thus, depth of field D_{TOT} increases linearly with decreasing sensor size
- this is why amateur cinematographers are drawn to SLRs
  - their chips are larger than even pro-level video camera chips
  - so they provide unprecedented control over depth of field

Vincent Laforet, Nocturne (2009)
Canon 1D Mark IV

Parting thought on DoF: the zen of *bokeh*



Canon 85mm
prime f/1.8 lens

- the appearance of sharp out-of-focus features in a photograph with shallow depth of field
  - determined by the shape of the aperture
  - people get religious about it
  - but not every picture with shallow DoF has evident bokeh...

Natasha Gelfand (Canon 100mm f/2.8 prime macro lens)

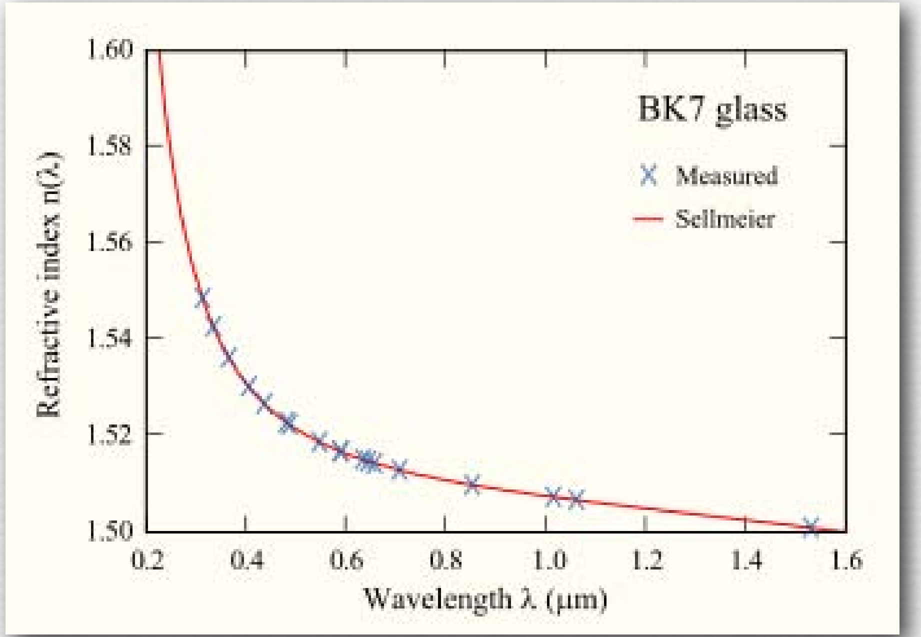
Lens aberrations

- chromatic aberrations
- Seidel aberrations, a.k.a. 3^{rd} order aberrations
  - arise because of error in our 1^{st} order approximation

$$\sin\phi \approx \phi \left(-\frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \frac{\phi^7}{7!} + \ldots \right)$$

  - spherical aberration
  - oblique aberrations
  - field curvature
  - distortion
  - can reduce all but distortion by closing down the aperture

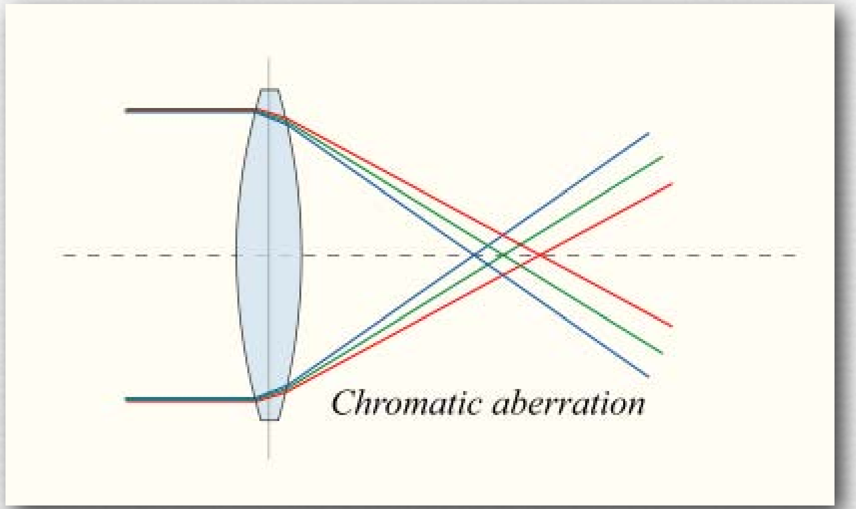
Dispersion

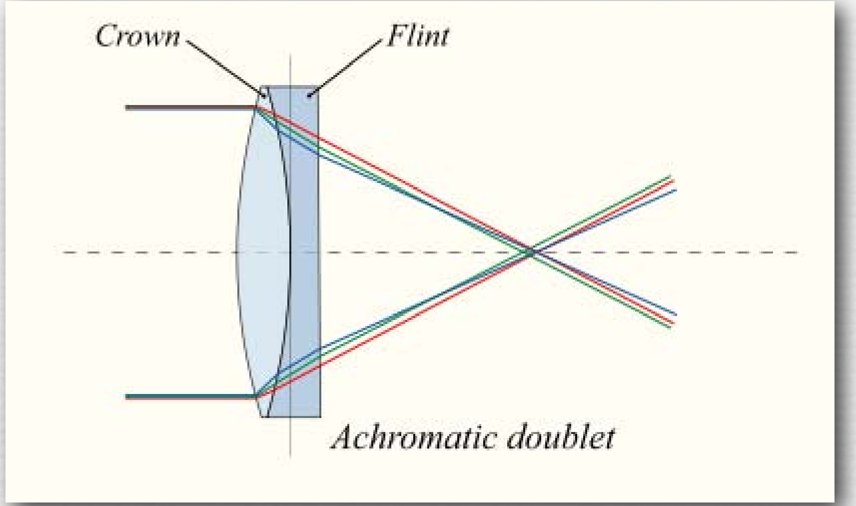


(wikipedia)

- index of refraction varies with wavelength
  - amount of variation depends on material
  - index is typically higher for blue than red
  - so blue light bends more

Chromatic aberration



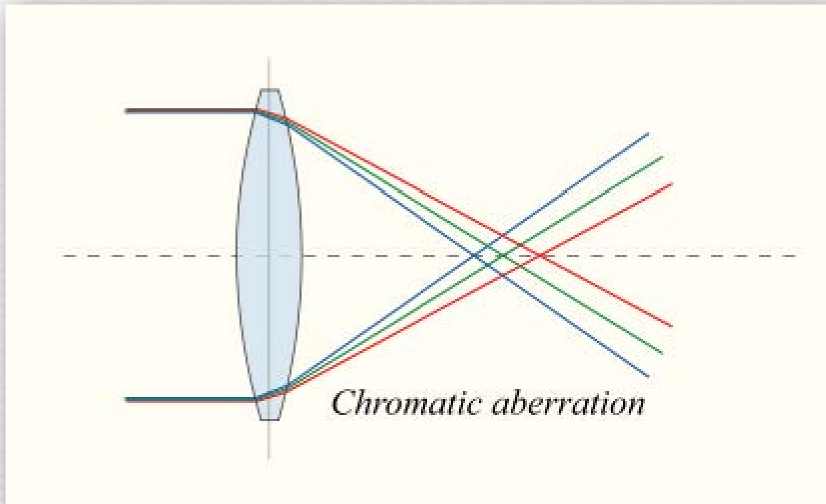


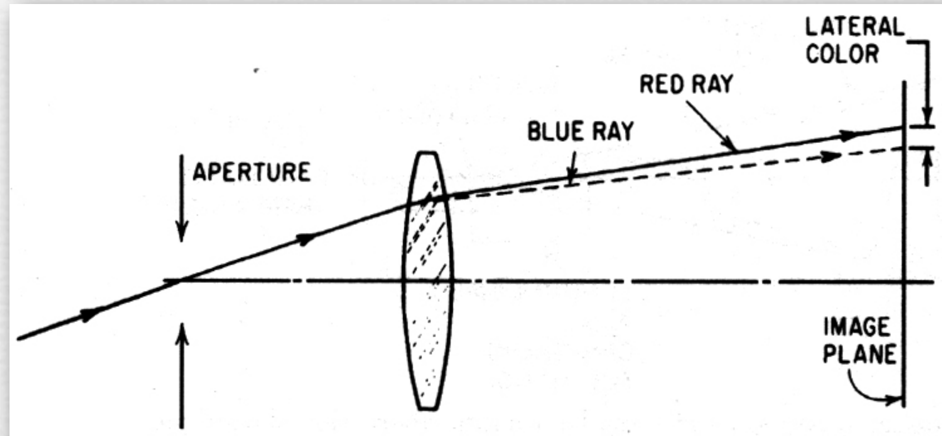
(wikipedia)

- dispersion causes focal length to vary with wavelength
  - for convex lens, blue focal length is shorter
- correct using *achromatic doublet*
  - low-dispersion positive lens + high-dispersion negative lens
  - can only correct at two wavelengths

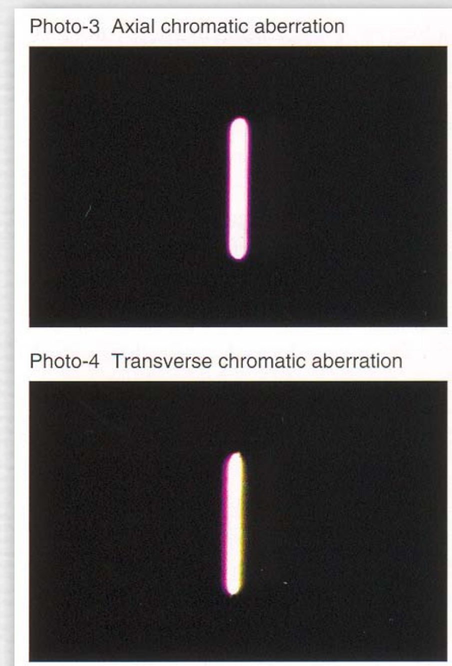
higher dispersion means more variation of *n* with wavelength

The chromatic aberrations





(Smith)



- change in focus with wavelength
  - called *longitudinal (axial) chromatic aberration*
  - appears everywhere in the image
- if blue image is closer to lens, it will also be smaller
  - called *lateral (transverse) chromatic aberration*
  - worse at edges of images than in center

Examples

- correctable in software
- not

(wikipedia)

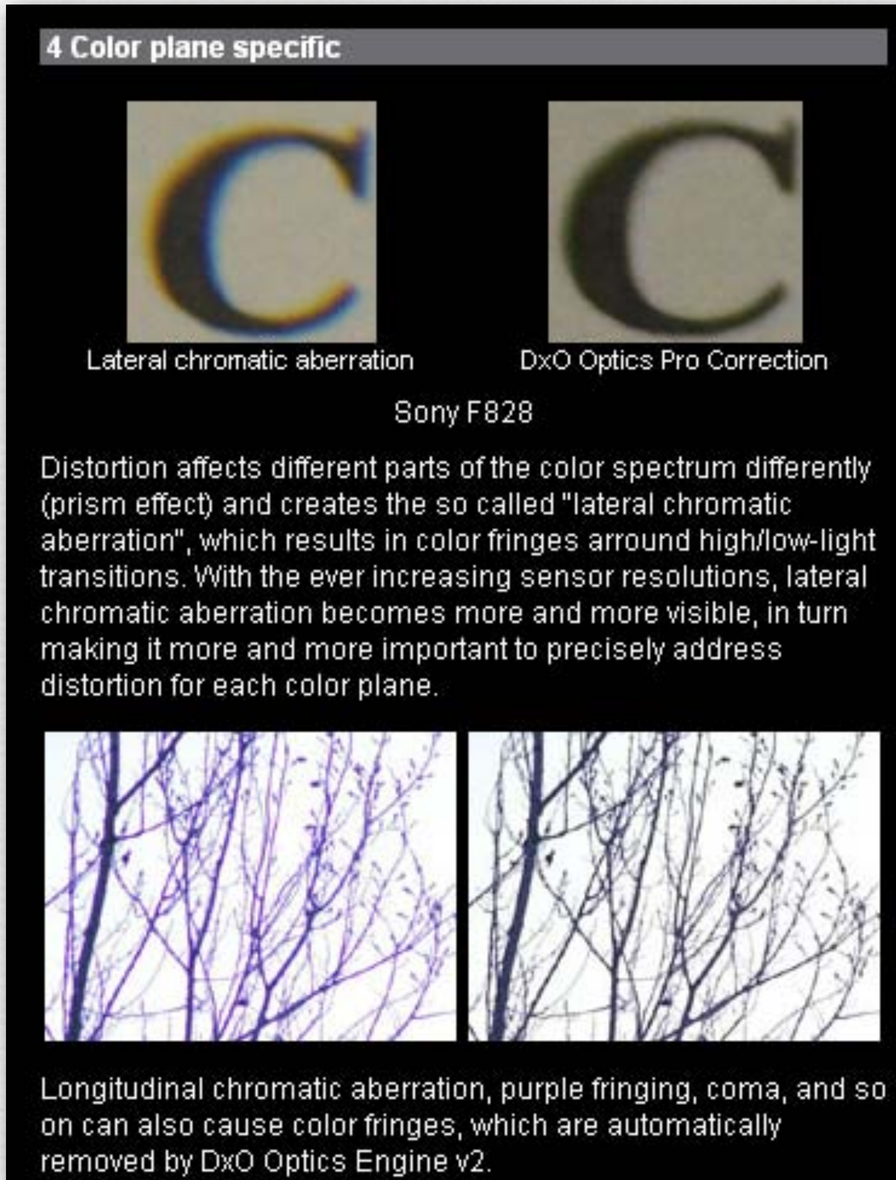
lateral

(toothwalker.org)

longitudinal

✦ other possible causes
  - demosiacing algorithm
  - per-pixel microlenses
  - lens flare

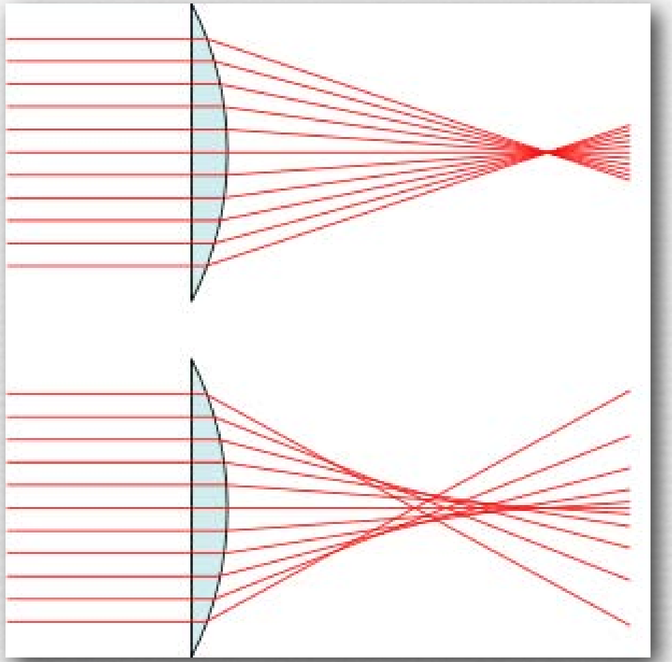
Software correction of lateral chromatic aberration



- Panasonic GF1 corrects for chromatic aberration in the camera (or in Adobe Camera Raw)
  - need focal length of lens, and focus setting

Q. Why don't humans see chromatic aberration?

Spherical aberration



(wikipedia)

- focus varies with ray height (distance from optical axis)
- can reduce by stopping down the aperture
- can correct using an aspherical lens
- can correct for this and chromatic aberration
  by combining with a concave lens of a different index

Examples

(Canon)

sharp

soft focus

Canon 135mm f/2.8 soft focus lens

Hubble telescope

before correction

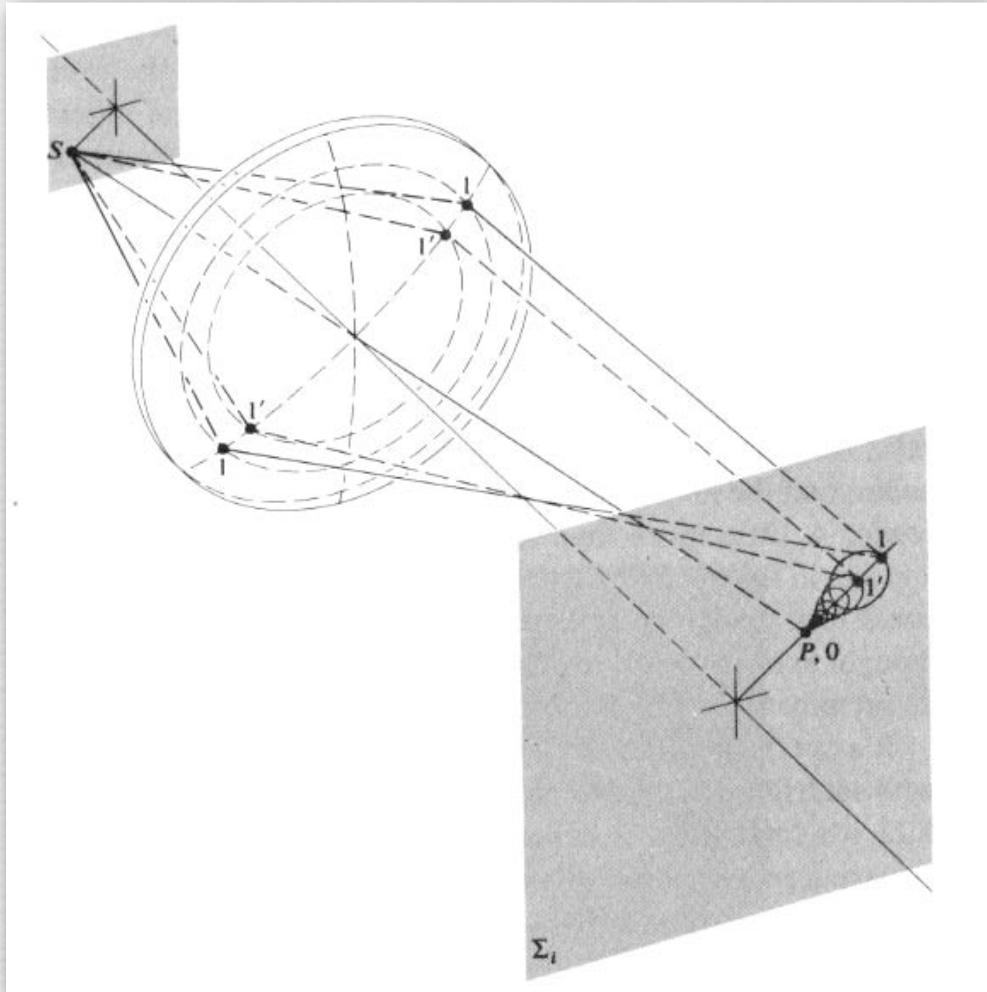
after correction

Oblique aberrations

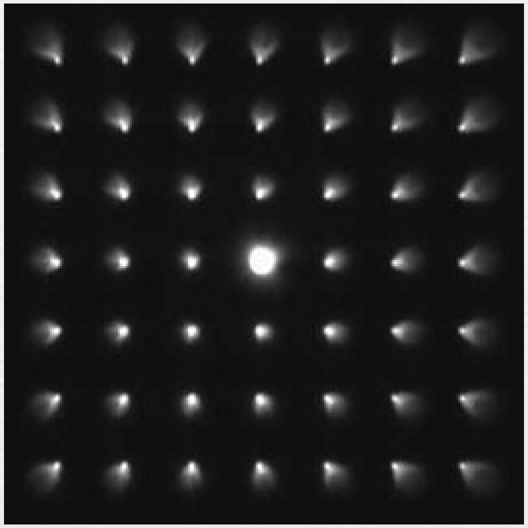
- spherical & chromatic aberrations occur on the optical axis, as well as off the axis
  - they appear everywhere in the field of view

- oblique aberrations do not appear in center of field
  - they get worse with increasing distance from the axis
  - coma and astigmatism

Coma



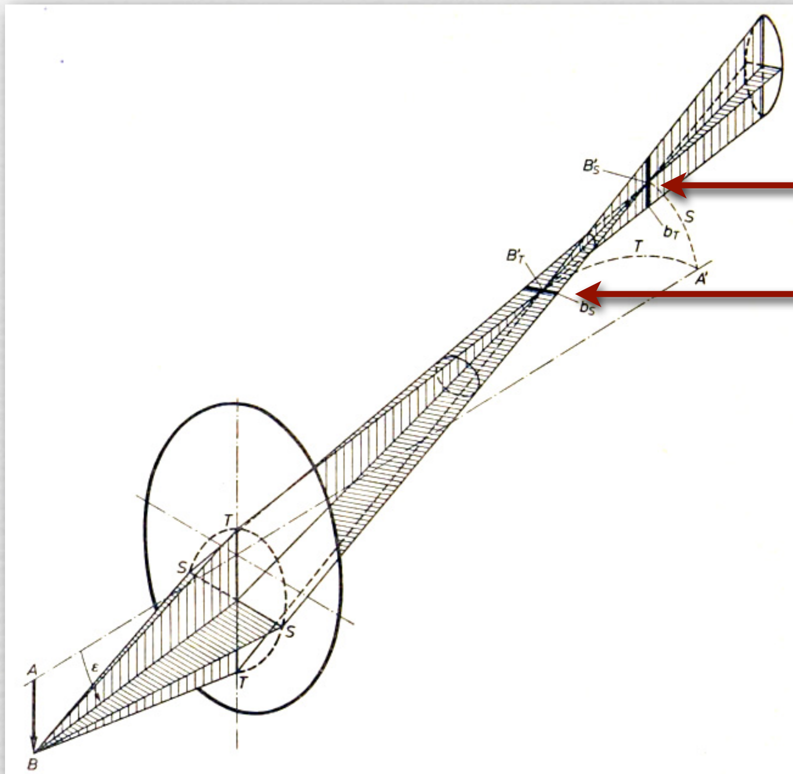
(Hecht)



(ryokosha.com)

- magnification varies with ray height (distance from optical axis)

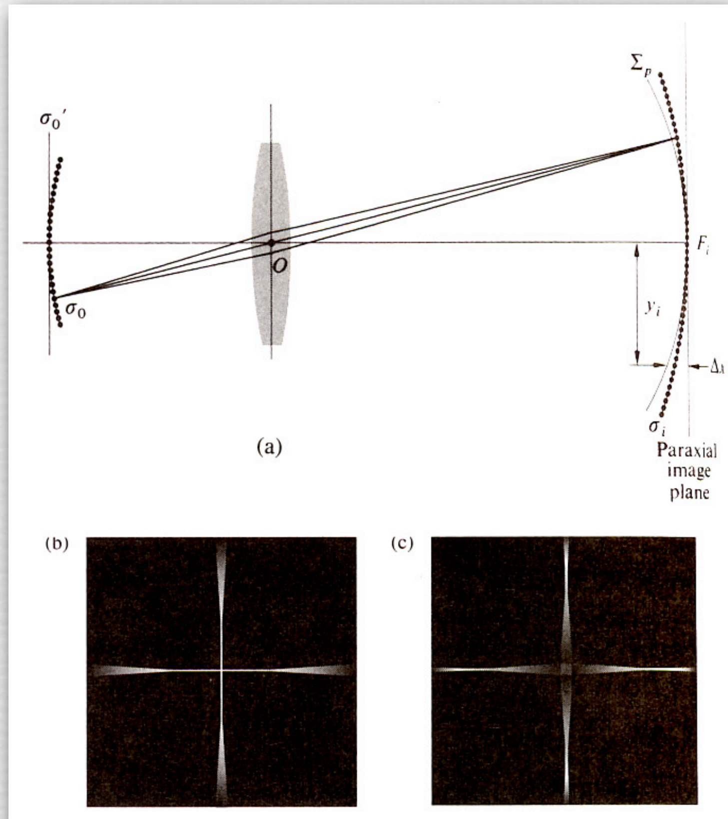
Astigmatism



(Pluta)

- ✦ tangential and sagittal rays focus at different depths
- ✦ my full eyeglass prescription
  - right: -0.75 -1.00 axis 135, left: -1.00 -0.75 axis 180

Field curvature

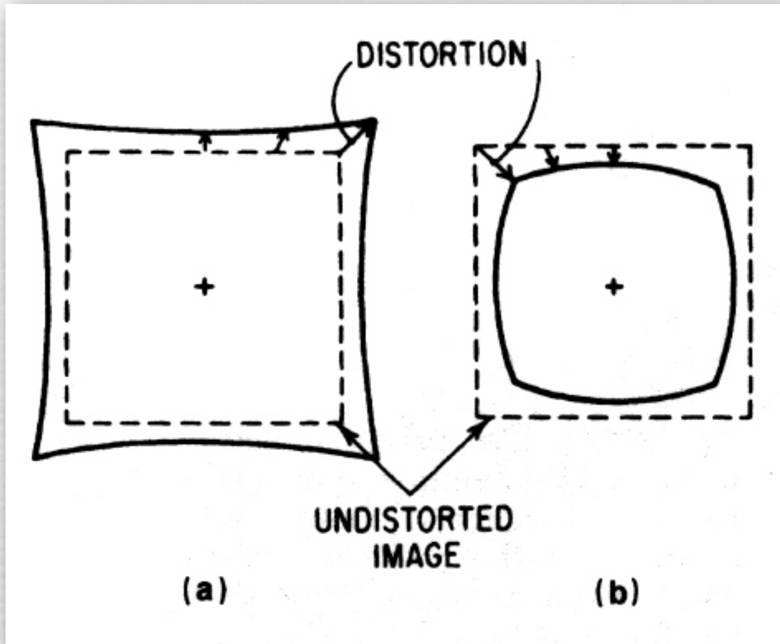


(Hecht)

- spherical lenses focus a curved surface in object space onto a curved surface in image space
- so a plane in object space cannot be everywhere in focus when imaged by a planar sensor

Distortion

(Smith)

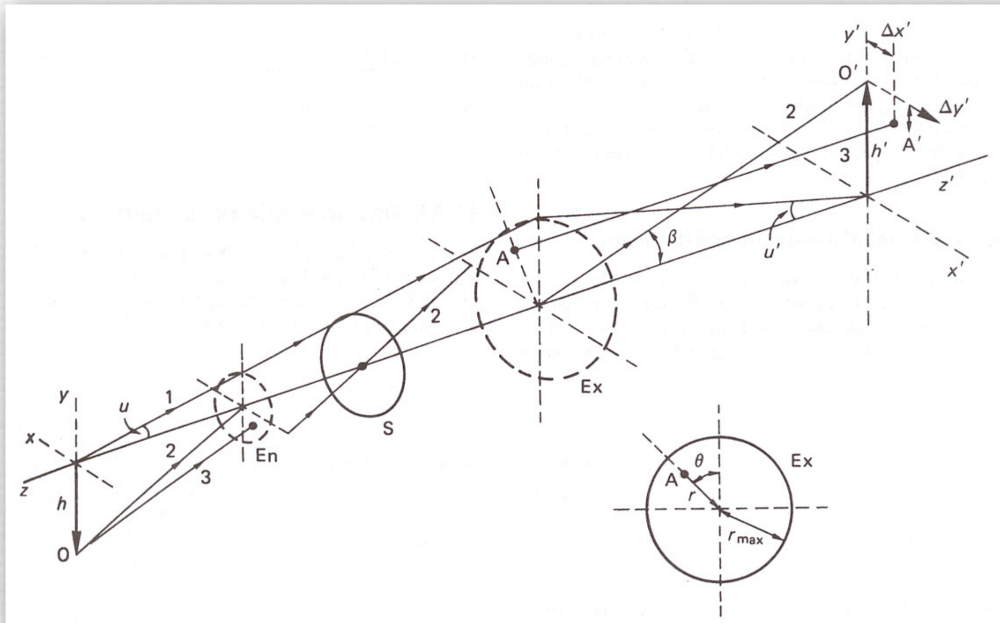


(Kingslake)

pincushion distortion

- change in magnification with image position
  (a) pincushion
  (b) barrel
- stopping down the aperture does not improve this

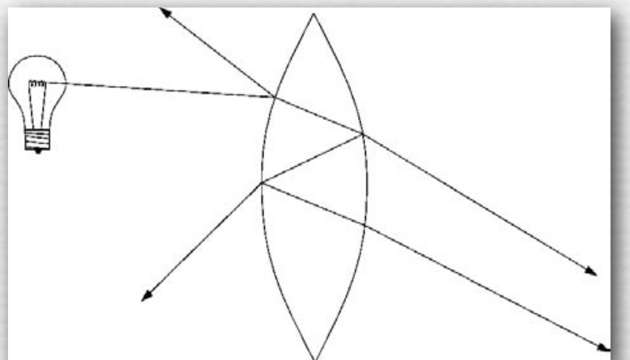
Algebraic formulation of monochromatic lens aberrations

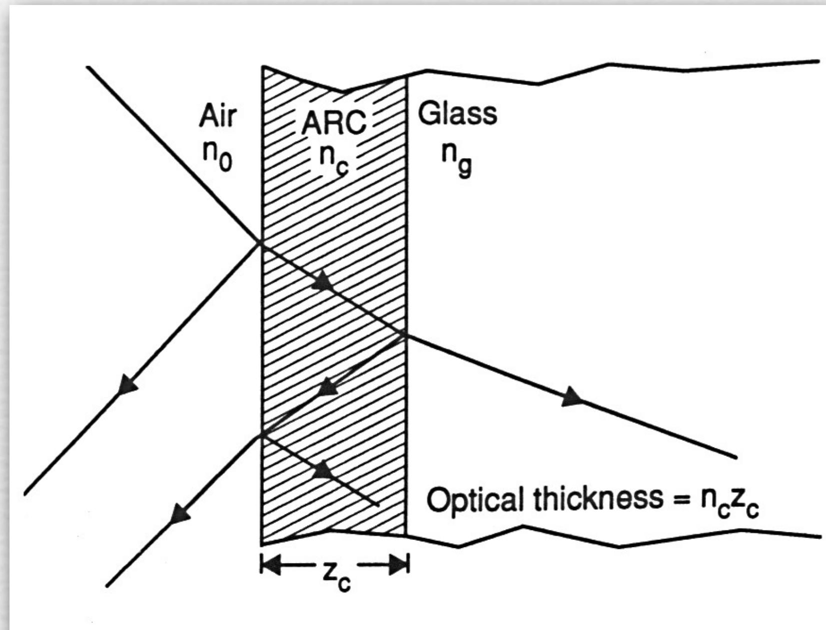


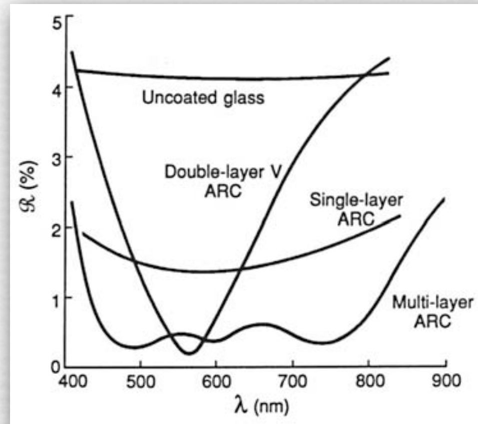
(Smith)

- spherical aberration $a_s r^4$
- coma $a_c h' r^3 \cos\theta$
- astigmatism $a_a h'^2 r^2 \cos^2\theta$
- field curvature $a_d h'^2 r^2$
- distortion $a_t h'^3 r \cos\theta$

Veiling glare



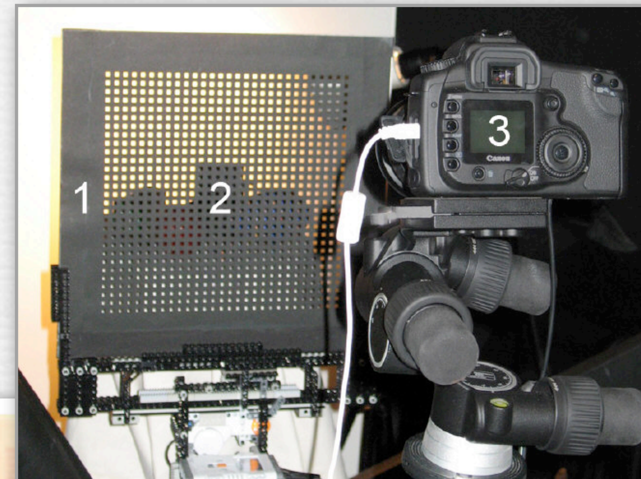




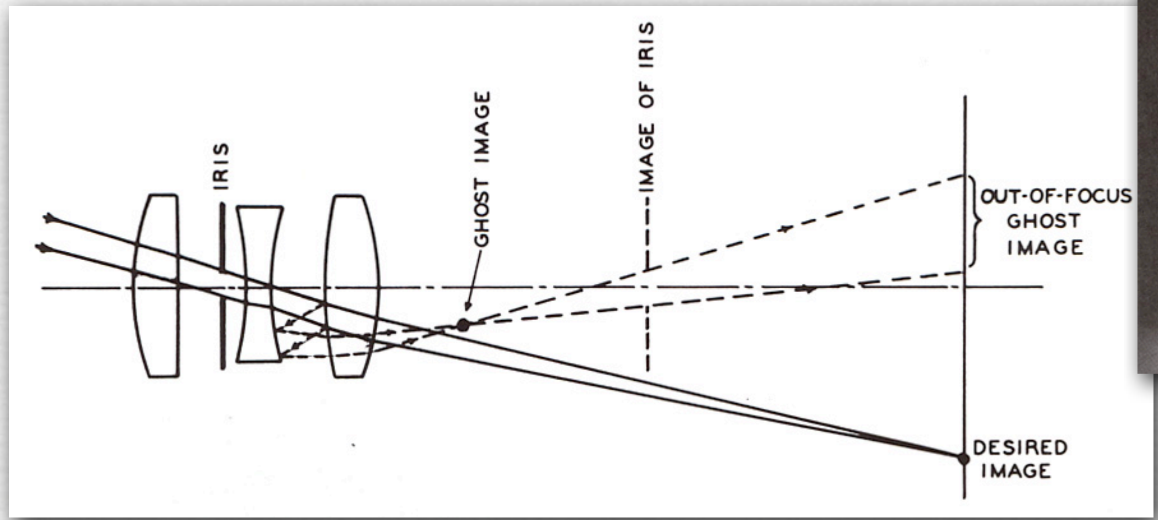
- contrast reduction caused by stray reflections
- can be reduced by anti-reflection coatings
  - based on interference, so optimized for one wavelength
  - to cover more wavelengths, use multiple coatings

Removing veiling glare computationally

[Talvala, Proc. SIGGRAPH 2007]



Flare and ghost images

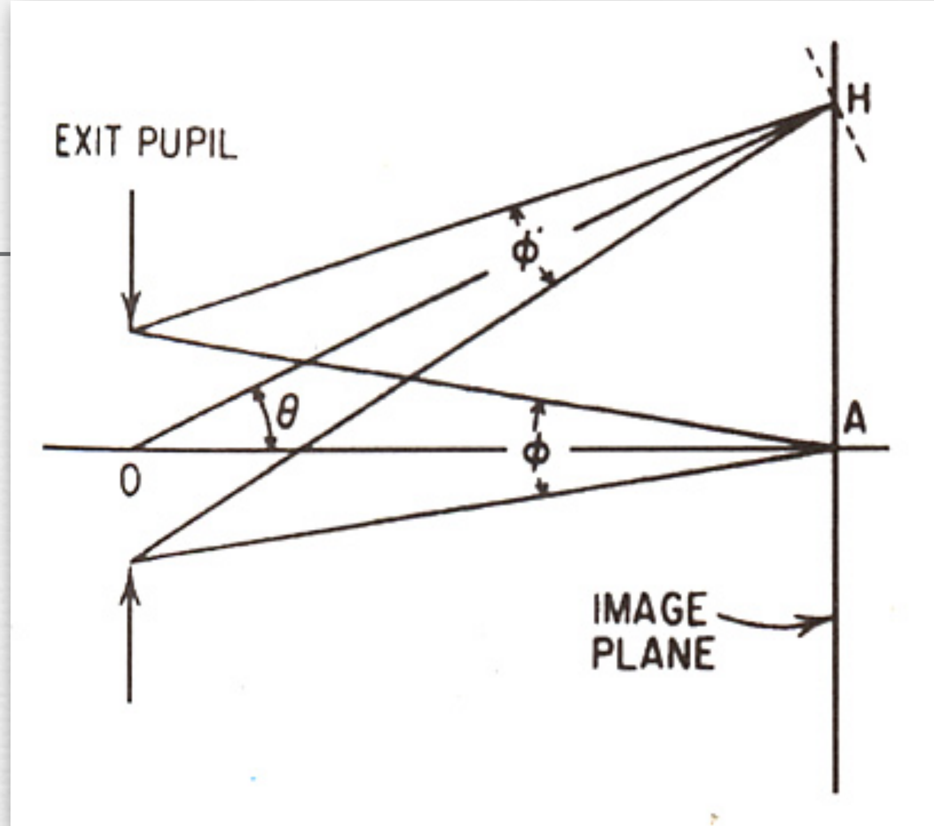


(Kingslake)

- reflections of the aperture, lens boundaries, etc., i.e. things inside the camera body
- removing these artifacts is an active area of research in computational photography
- but it's a hard problem

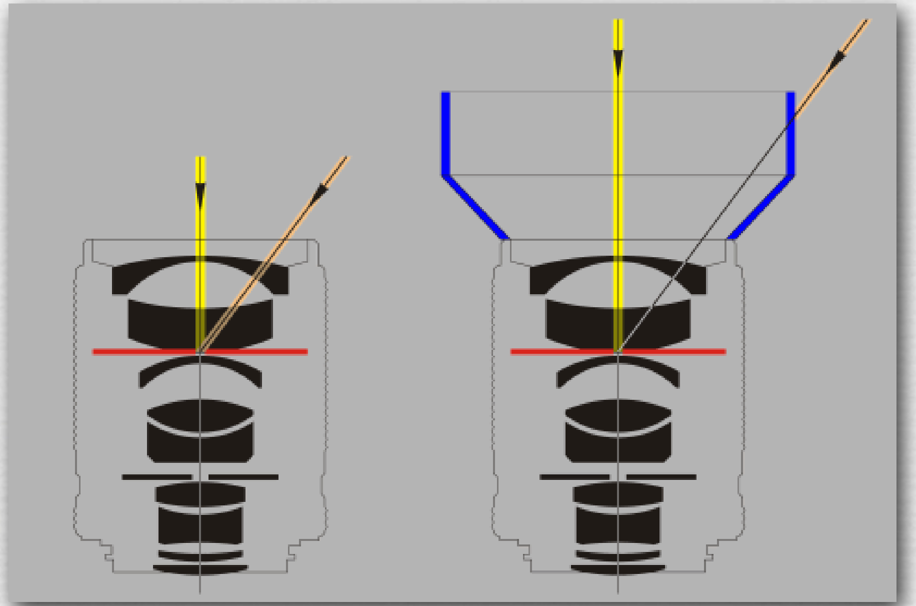
Vignetting
(a.k.a. **natural** vignetting)

(Smith)



- irradiance is proportional to projected area of aperture as seen from pixel on sensor, which drops as $\cos\theta$
- irradiance is proportional to projected area of pixel as seen from aperture, which also drops as $\cos\theta$
- irradiance is proportional to distance^2 from aperture to pixel, which rises as $1/\cos\theta$
- combining all these effects, light drops as $\cos^4\theta$

Other sources of vignetting



(toothwalker.org)

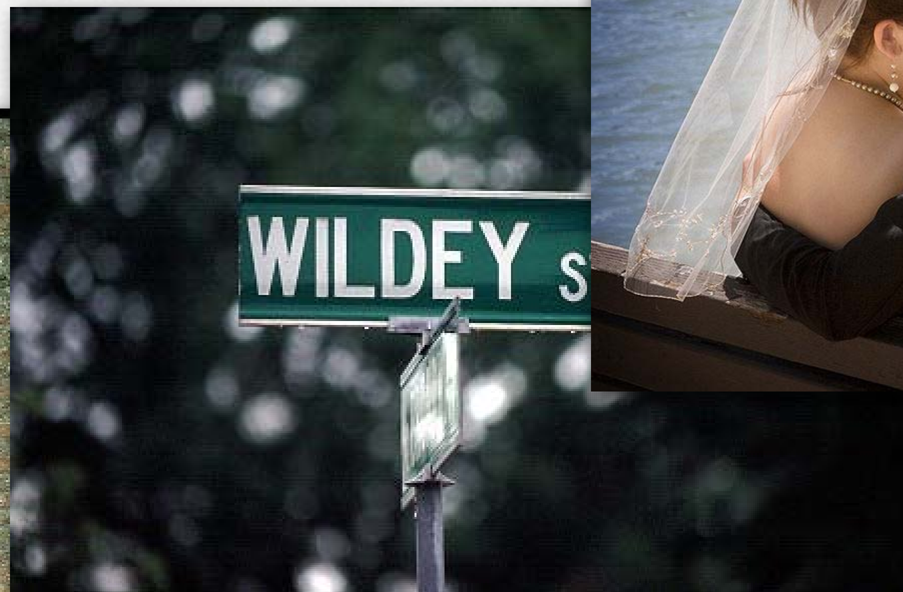
optical vignetting
from multiple lens elements,
especially at wide apertures

mechanical vignetting
from add-on lens hoods
(or filters or fingers)

- **pixel** vignetting due to shadowing inside each pixel
(we'll come back to this)

Examples

(wikipedia)

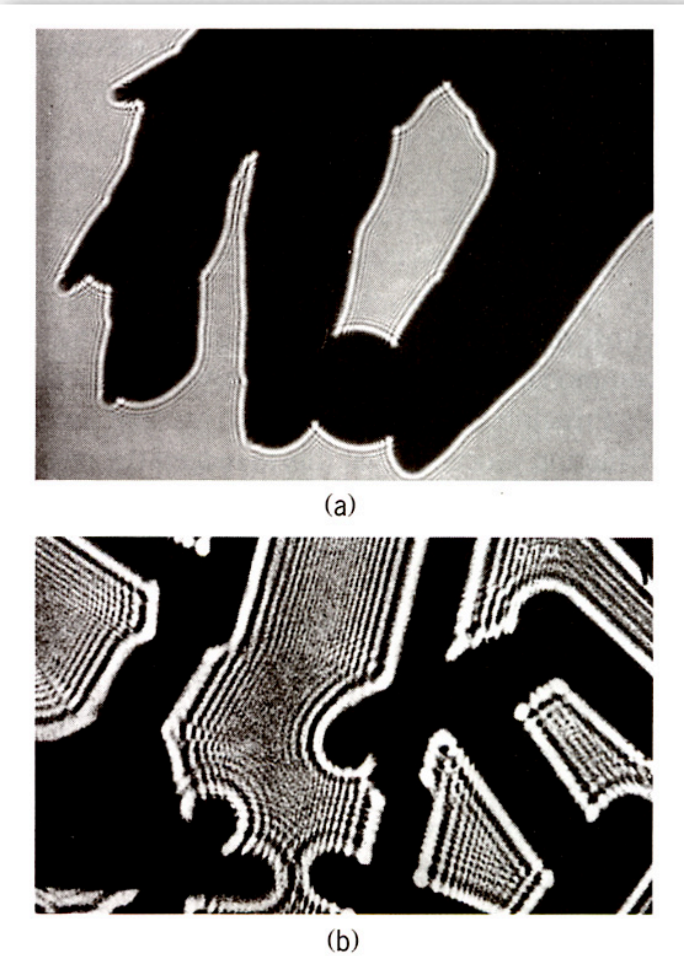


(toothwalker.org)

(toothwalker.org)

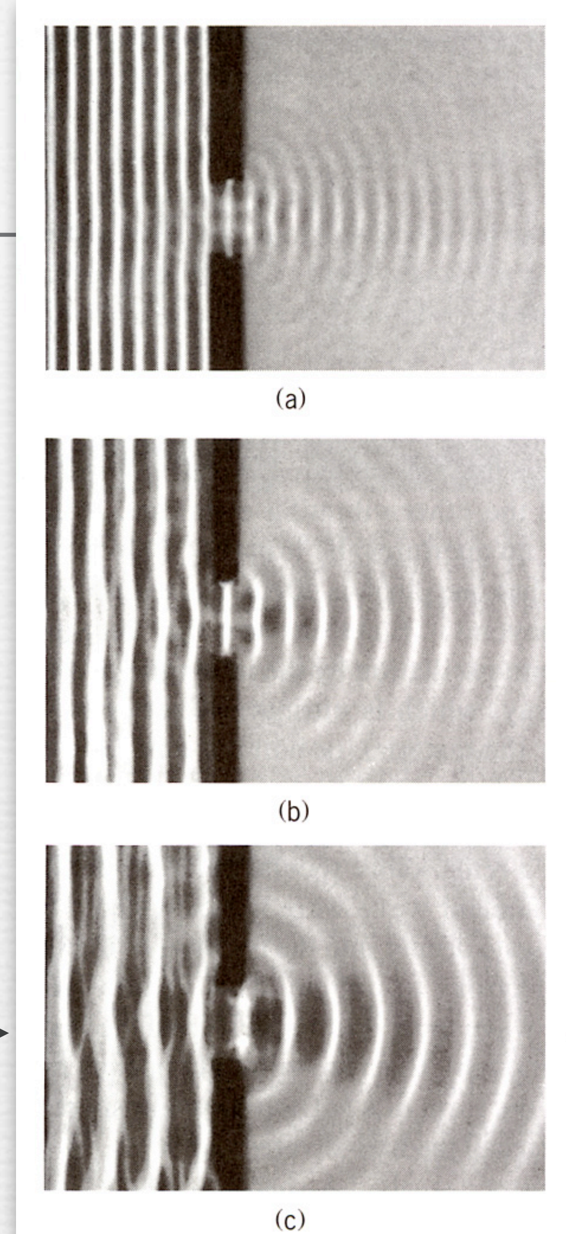
- vignetting affects the *bokeh* of out-of-focus features
- vignetting is correctable in software,
  but boosting pixel values worsens noise
- vignetting can be appled afterwards, for artistic purposes

Diffraction



illuminated by a (spread-out) laser beam & recorded directly on film

(Hecht)



varying the wavelength of waves passing through a slit in a ripple tank

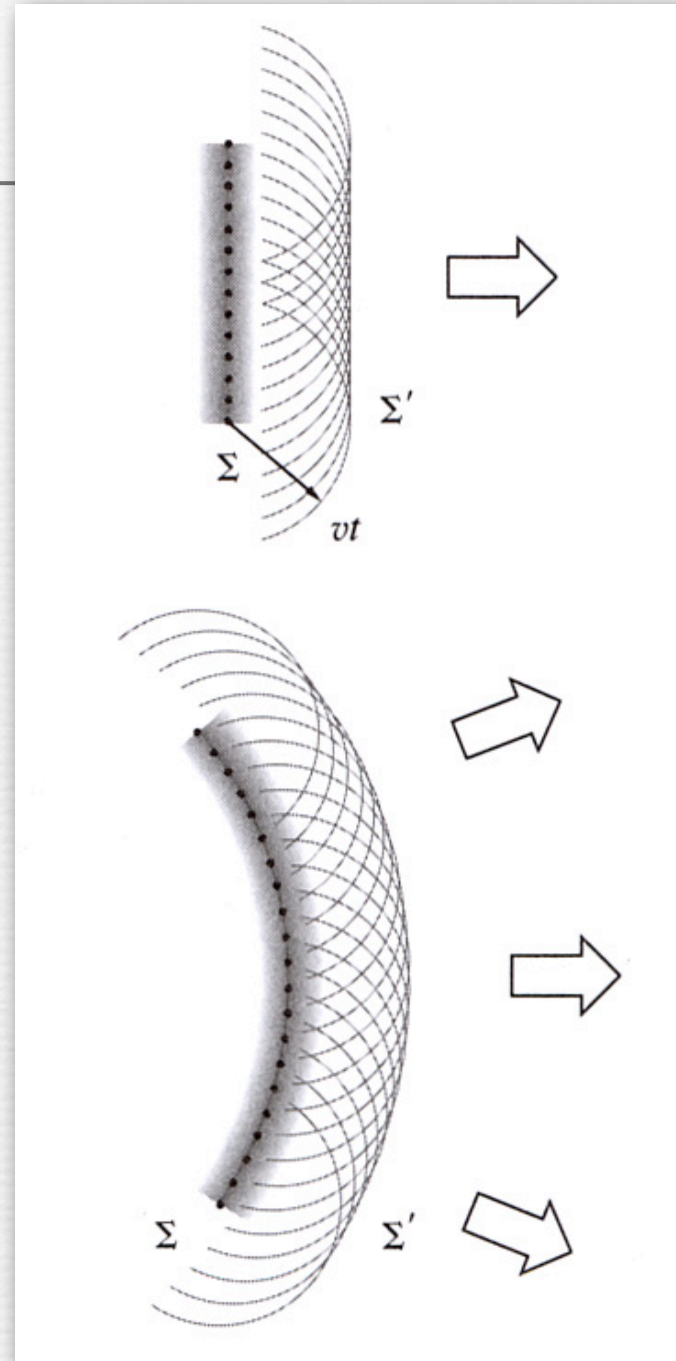
- as wavelength decreases in the ripple tank, propagation becomes more ray-like

Huygens wavelets

- every point on a wavefront can be considered as a source of spherical wavelets

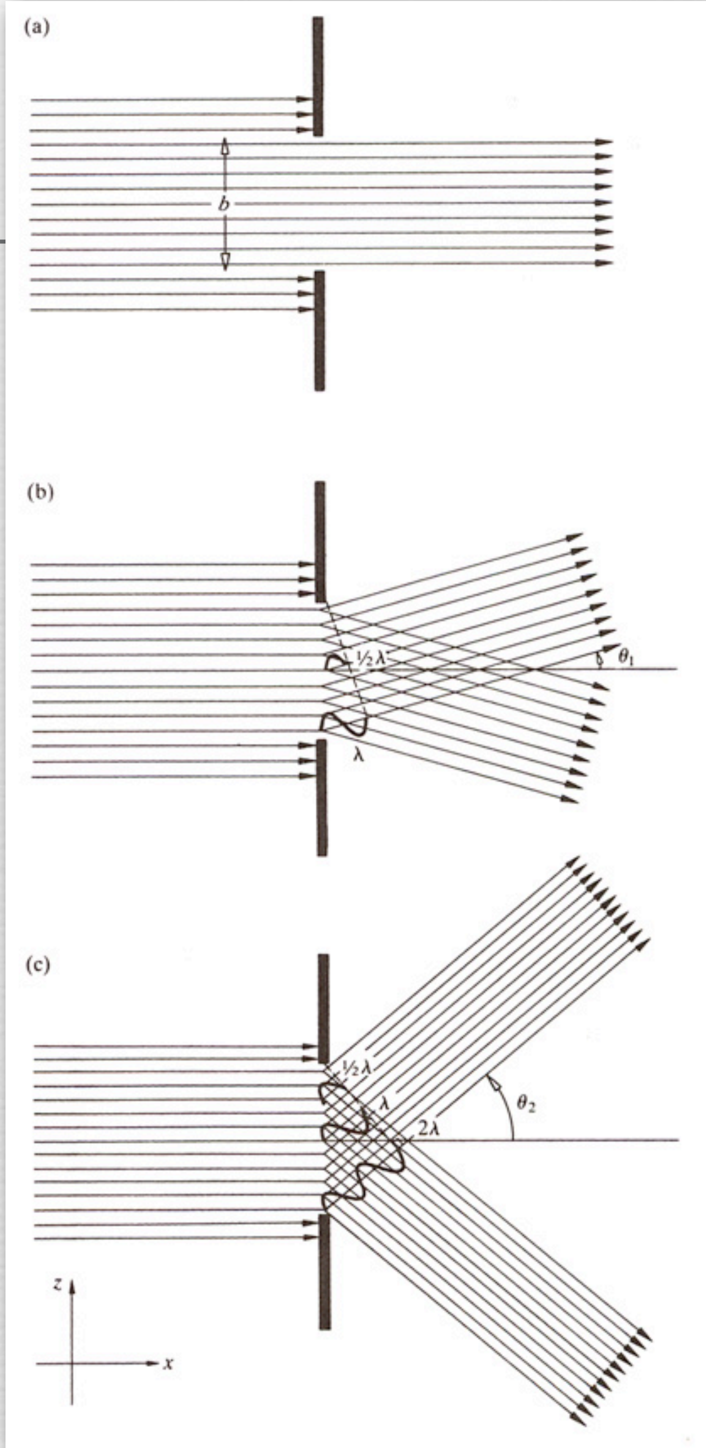
- the optical field is the superimposition of these waves, after allowing for constructive or destructive interference

(Hecht)



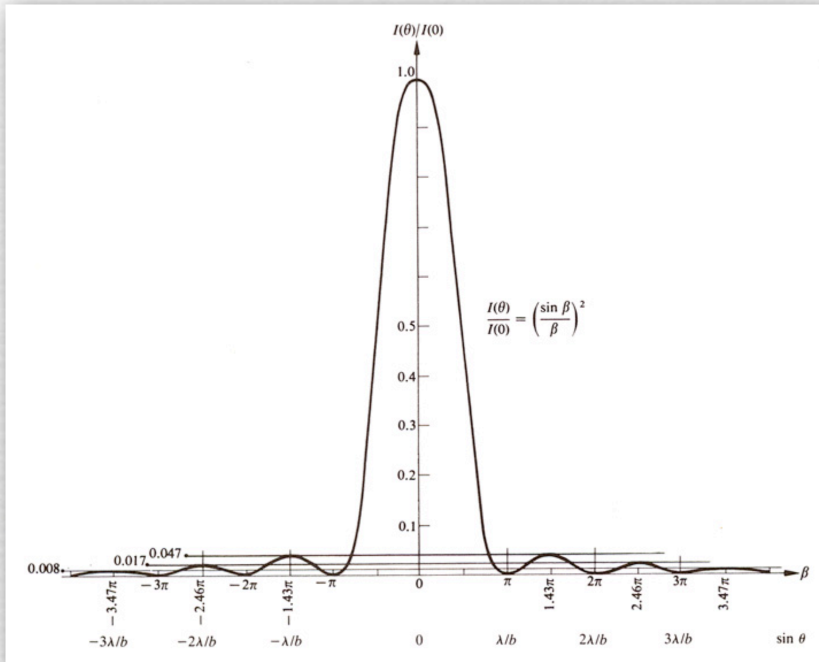
Diffraction from a slit

- rays leaving the slit and traveling perpendicularly (a) have the same phase at each distance from the slit, so they add, producing a maximum
- for rays traveling in the direction θ_1 (b), waves of all phases from 0° to 360° are present; these cancel, producing a minimum (black)
- at a greater angle (not shown), waves of phases from 0° to e.g. 540° are present; not all are canceled, producing a second maximum
- in the direction θ_2 (c) waves cancel again, producing black



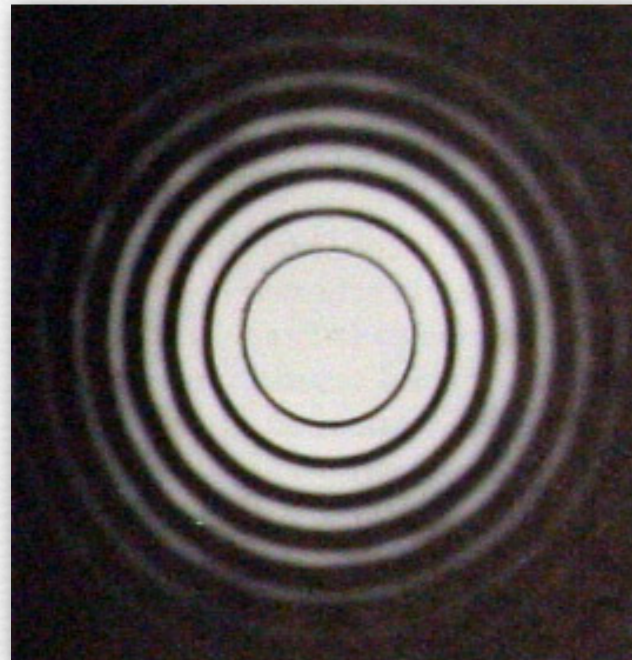
(Hecht)

Frauenhofer diffraction



$$\frac{I(\theta)}{I(0)} = \left(\frac{\sin\beta}{\beta}\right)^2$$

diffraction from a slit



(Hecht)

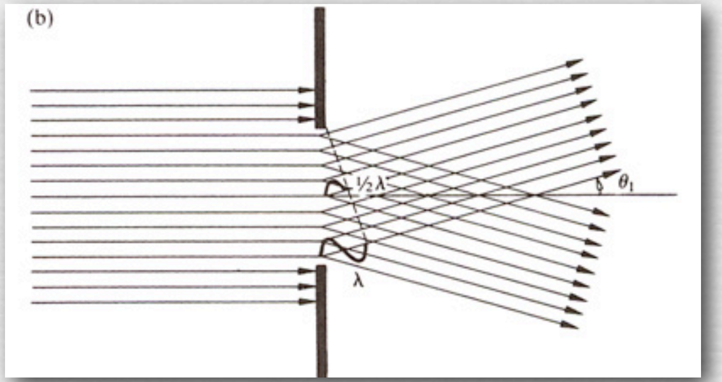
diffraction from a circular aperture: Airy rings

- diffraction viewed from a long distance ("far field")

Diffraction in photographic cameras

- well-corrected lenses are called *diffraction-limited*

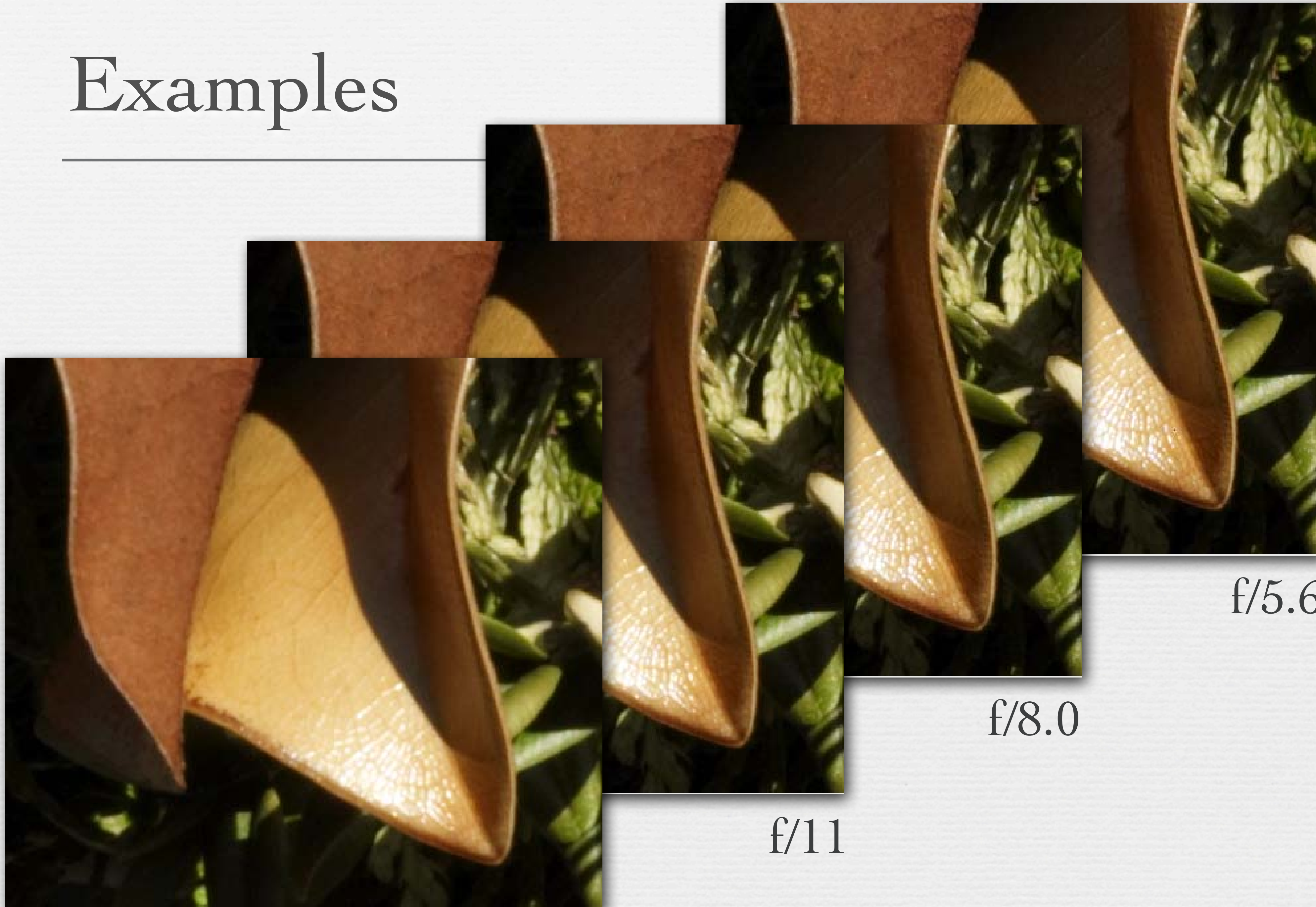
- the smaller the aperture (A), the larger the diffraction blur
  - as the aperture shrinks, angle θ must be greater before all phases from 0° to 360° are present, producing the first black ring; this spreads out the Airy pattern



- the longer the distance to the sensor (f), the larger the blur
  - the Airy pattern continues to spread spatially as it propagates

- thus, the size of the blur varies with $N = f / A$

Examples



f/5.6

f/8.0

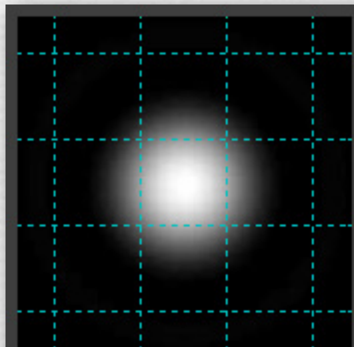
f/11

f/22

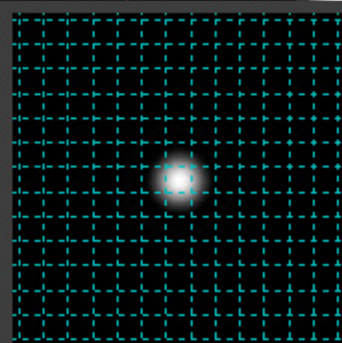
(luminous-landscape.com)

Diffraction in photographic cameras

- the smaller the pixels, the more of them the pattern covers
  - if the pattern spans >> 1 pixel, the image becomes blurry



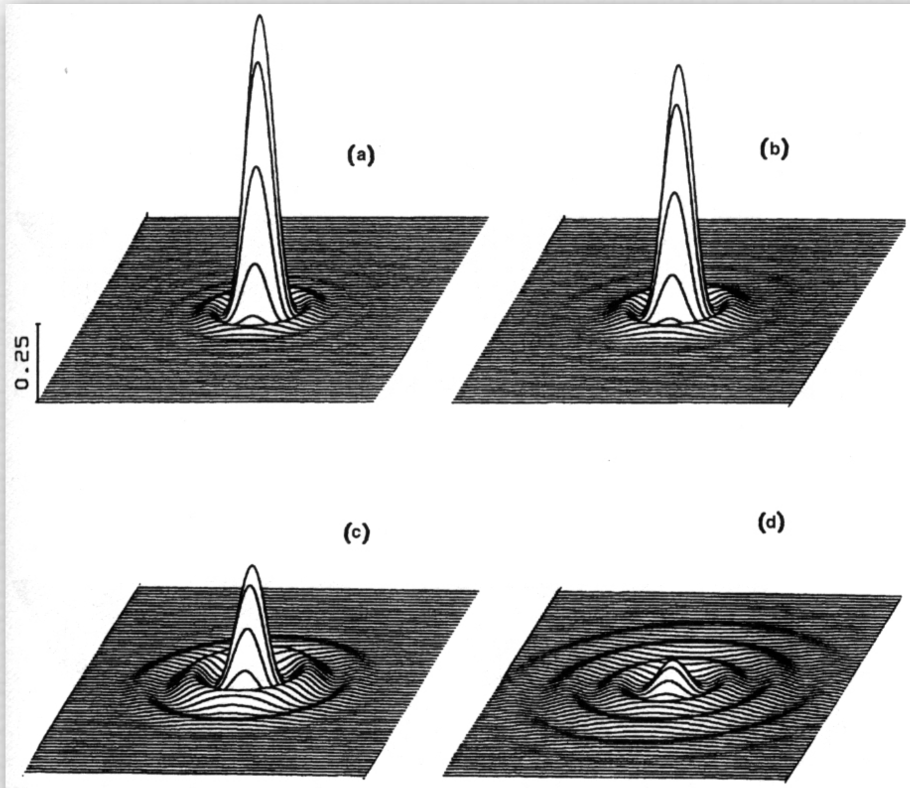
Aperture	Camera Type	Pixel Area
f/2.0	Canon EOS 1D	136. µm²
f/2.8	Canon EOS 1Ds	77.6 µm²
f/4.0	Canon EOS 1DMkII / 5D	67.1 µm²
f/5.6	Nikon D70	61.1 µm²
f/8.0	Canon EOS 10D	
f/11	Canon EOS 1DsMkII	
f/16	Canon EOS 20D / 35	
f/22	Nikon D2X	
f/32	Canon PowerShot G(	



Aperture	Camera Type	Pixel Area
f/2.0	Canon EOS 1D	136. µm²
f/2.8	Canon EOS 1Ds	77.6 µm²
f/4.0	Canon EOS 1DMkII / 5D	67.1 µm²
f/5.6	Nikon D70	61.1 µm²
f/8.0	Canon EOS 10D	54.6 µm²
f/11	Canon EOS 1DsMkII	52.0 µm²
f/16	Canon EOS 20D / 350D	41.2 µm²
f/22	Nikon D2X	30.9 µm²
f/32	Canon PowerShot G6	5.46 µm²

(http://www.cambridgeincolour.com/tutorials/diffraction-photography.htm)

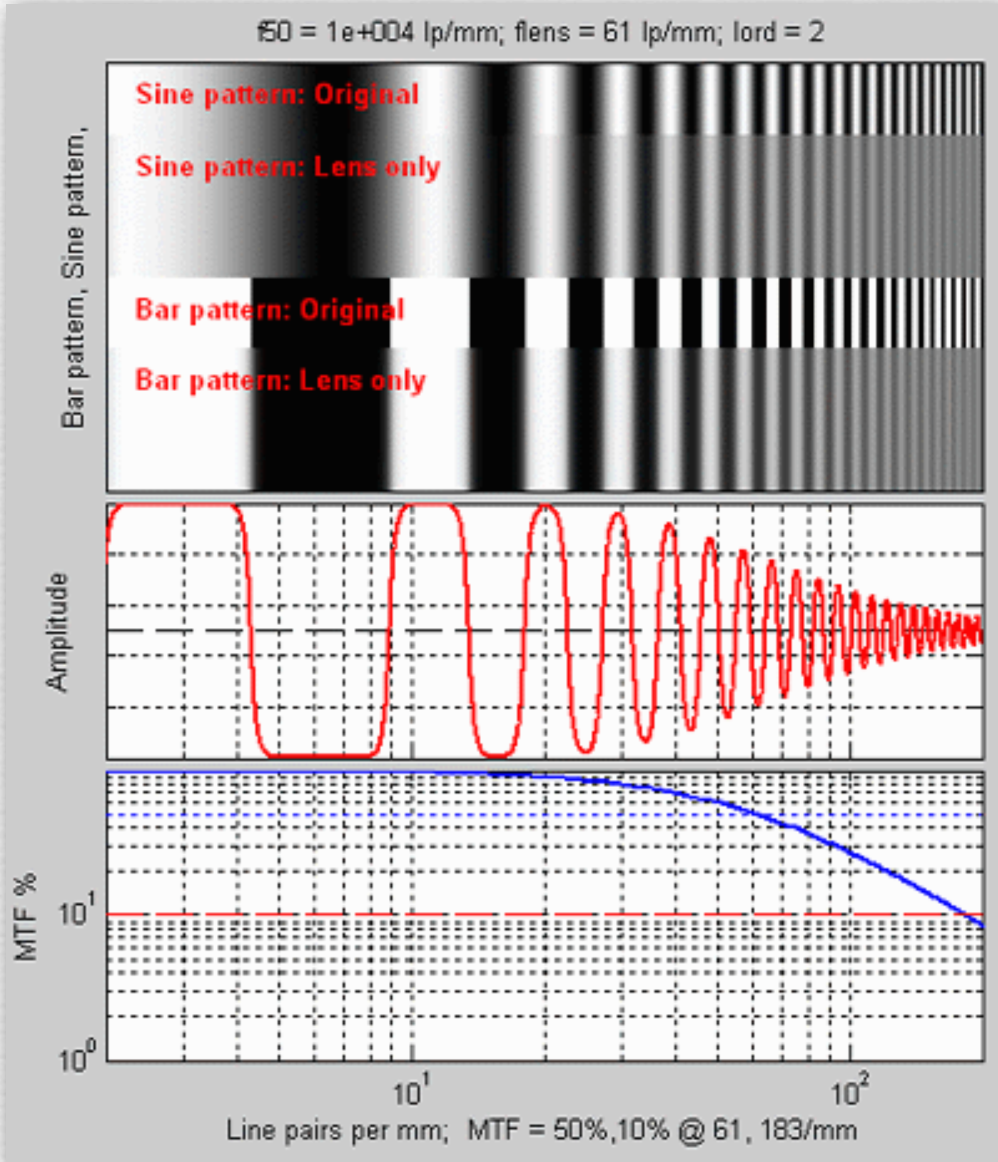
Describing sharpness: the point spread function (PSF)



(Smith)

- the image of a point source as the amount of spherical aberration in the optical system is gradually increased
- combines blur due to aberration and diffraction effects

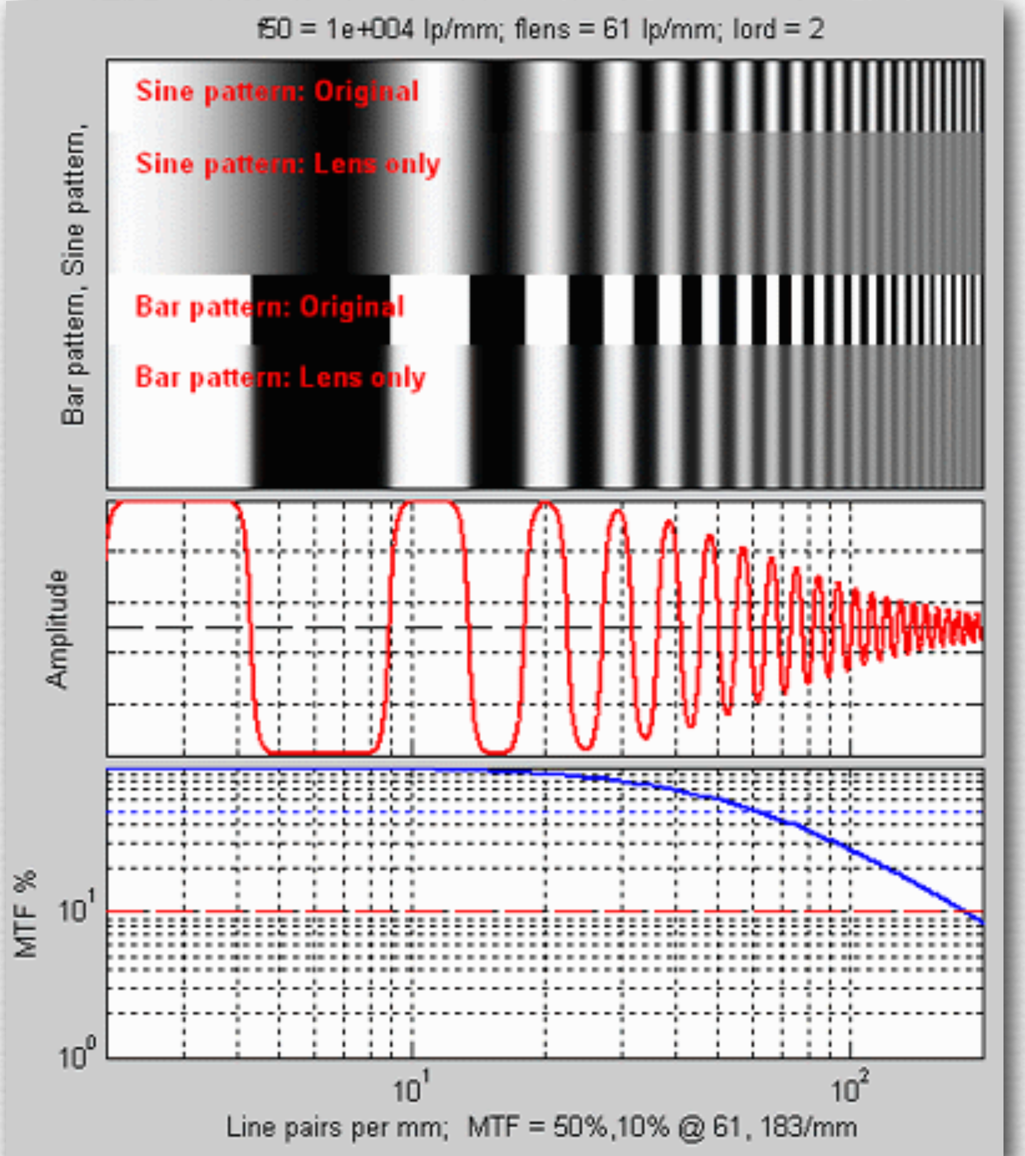
Describing sharpness: the modulation transfer function (MTF)



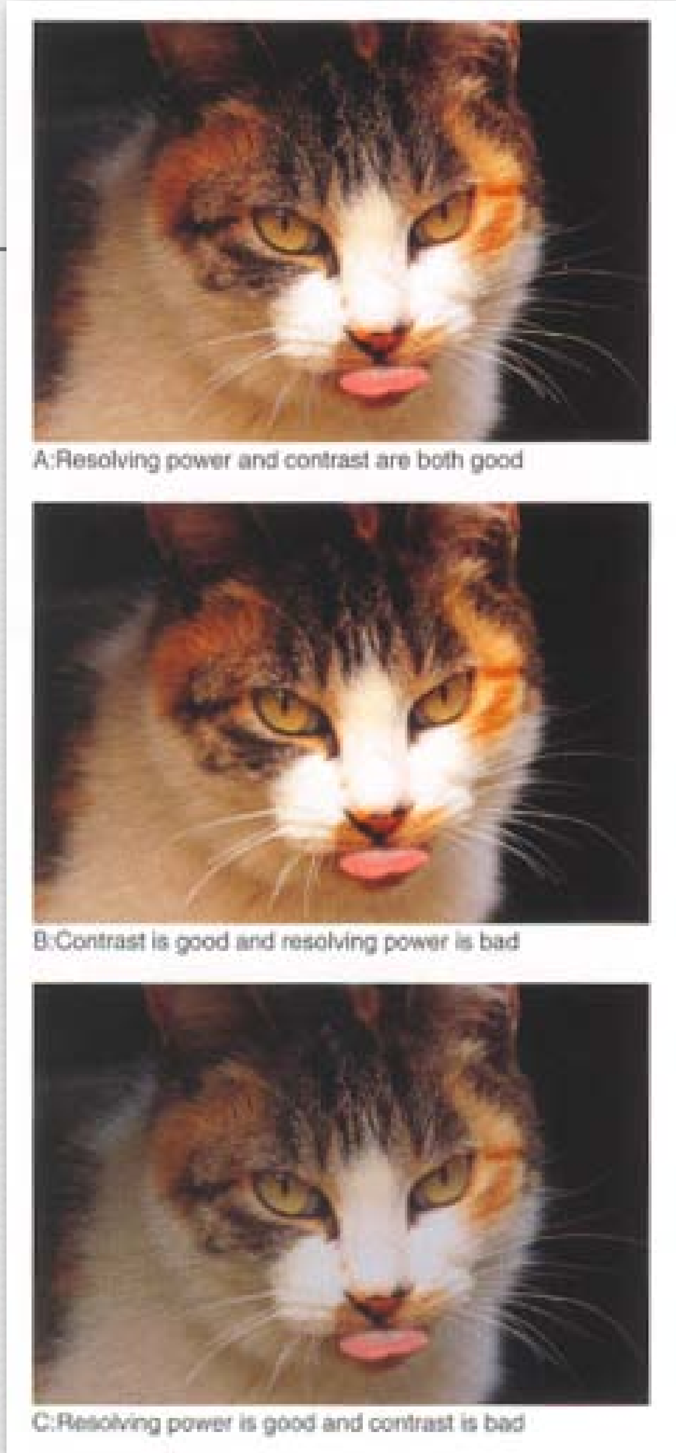
- ✦ the amount of each spatial frequency that can be reproduced by an optical system

(imatest.com)

Sharpness versus contrast

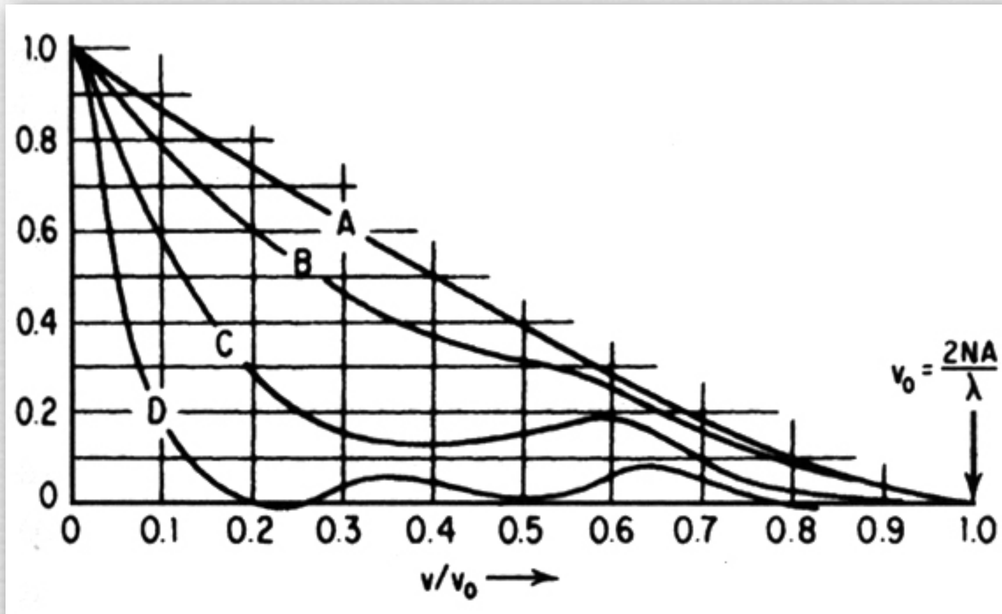


(imatest.com)



(Canon)

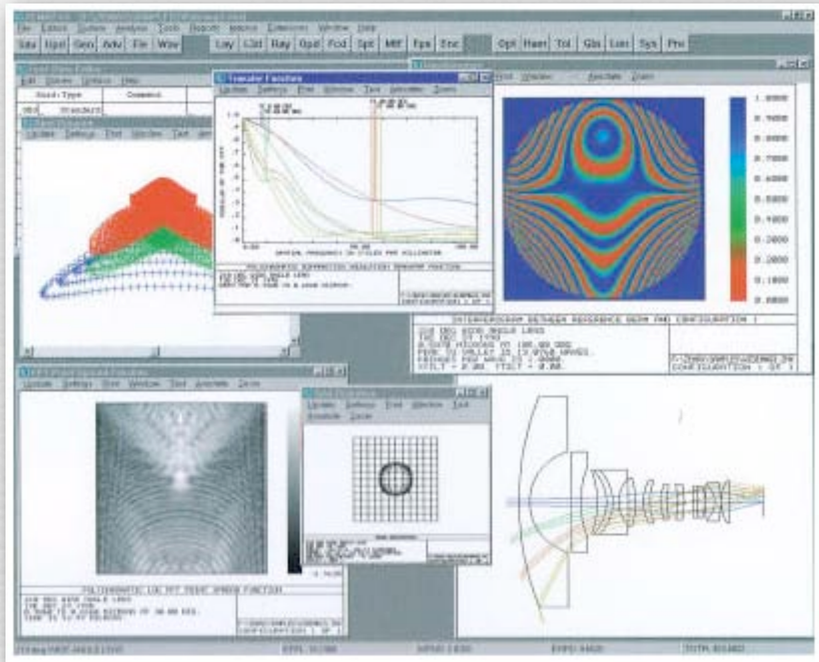
MTF curves

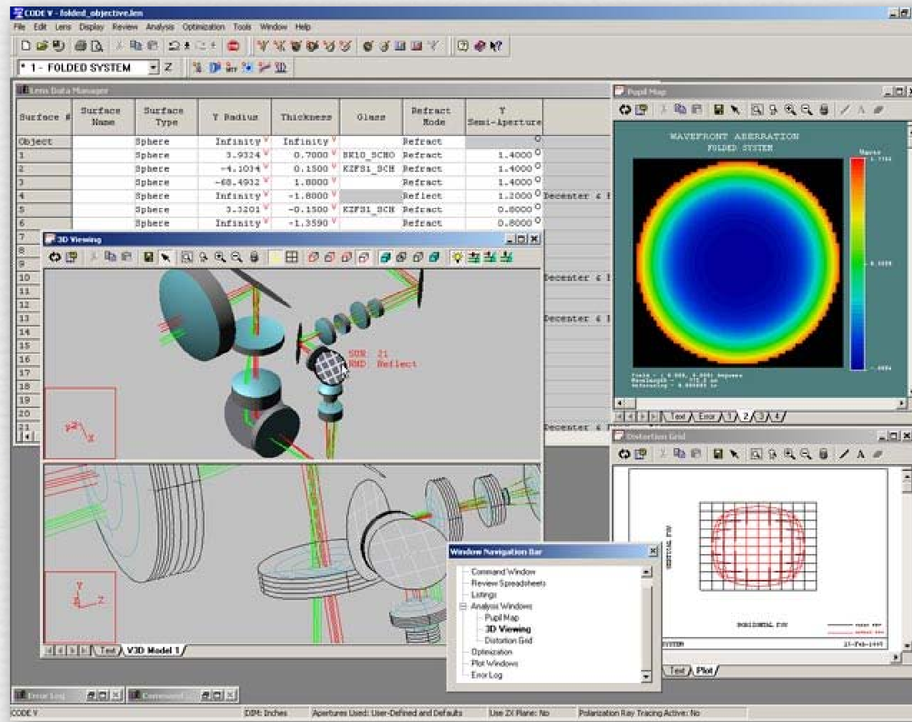


(Smith)

- the amount of each spatial frequency that can be reproduced by a diffraction-limited optical system
- A-D represent different amounts of defocus
- the cutoff at right is the diffraction limit for a given aperture (NA ≈ 1/2N) and wavelength (λ)

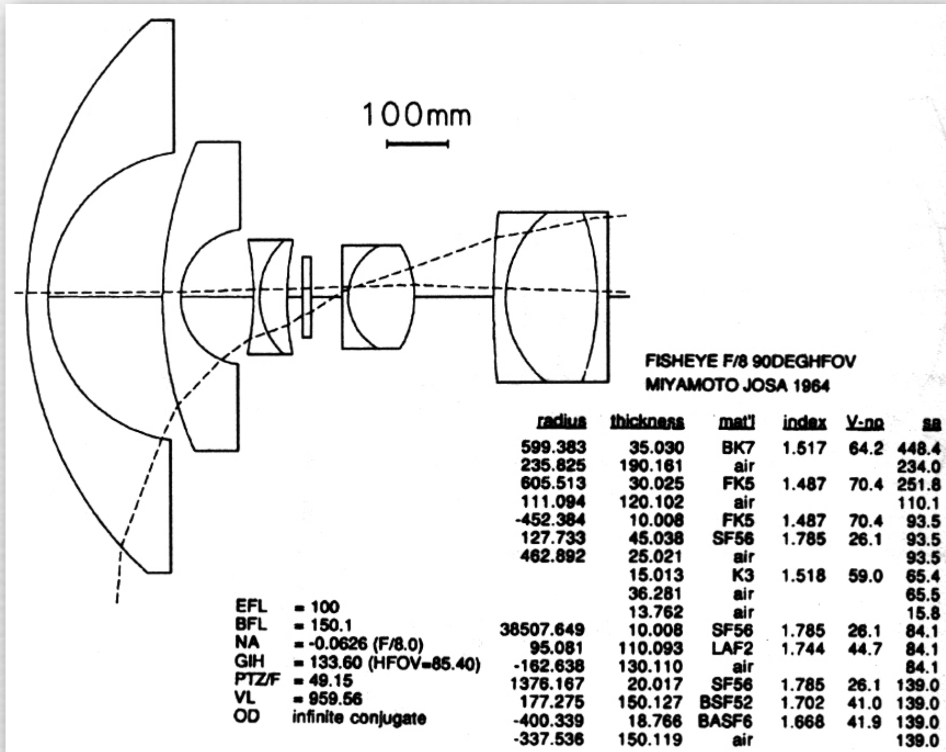
Lens design software



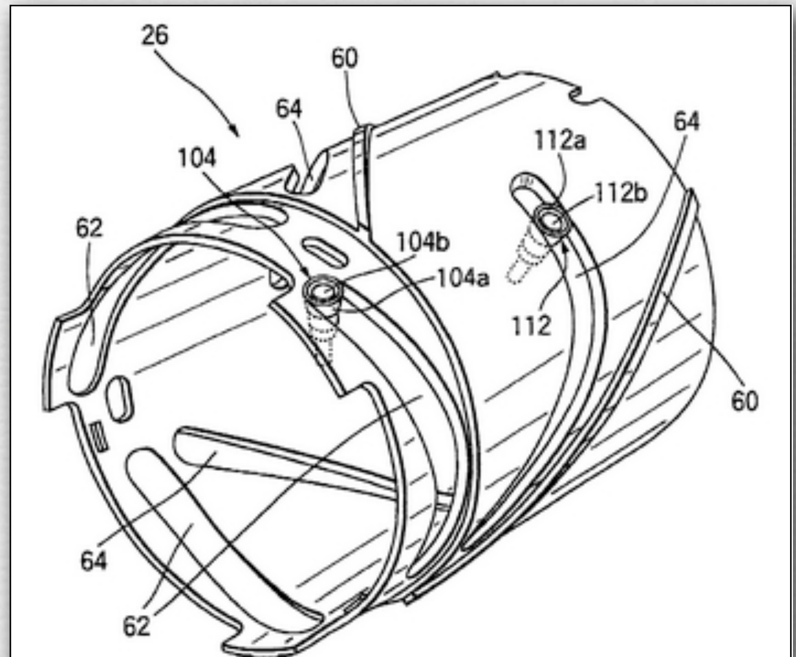


✦ uses optimization to make good recipes better

Lens catalogs and patents



radius	thickness	mat'l	index	V-no	sa
599.383	35.030	BK7	1.517	64.2	448.4
235.825	190.161	air			234.0
605.513	30.025	FK5	1.487	70.4	251.8
111.094	120.102	air			110.1
-452.384	10.008	FK5	1.487	70.4	93.5
127.733	45.038	SF56	1.785	26.1	93.5
462.892	25.021	air			93.5
	15.013	K3	1.518	59.0	65.4
	36.281	air			65.5
	13.762	air			15.8
38507.649	10.008	SF56	1.785	26.1	84.1
95.081	110.093	LAF2	1.744	44.7	84.1
-162.638	130.110	air			84.1
1376.167	20.017	SF56	1.785	26.1	139.0
177.275	150.127	BSF52	1.702	41.0	139.0
-400.339	18.766	BASF6	1.668	41.9	139.0
-337.536	150.119	air			139.0



- ✦ hard to find optical recipe for commercial camera lenses

DoF and the dolly-zoom

- if we zoom in (change f)
  and stand further back (change U) by the same factor

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

- the depth of field stays the same!
  - useful for macro when you can't get close enough

50mm f/4.8

(juzaphoto.com)

200mm f/4.8,
moved back 4× from subject

Slide credits

- Steve Marscher
- Fredo Durand

- Cole, A., *Perspective*, Dorling Kindersley, 1992.
- Kemp, M.,*The Science of Art*,Yale University Press, 1990.
- Hecht, E., *Optics* (4th ed.), Pearson / Addison-Wesley, 2002.
- Renner, E., *Pinhole Photography* (2nd ed.), Focal Press, 2000.
- London, Stone, and Upton, *Photography* (9th ed.), Prentice Hall, 2008.
- D'Amelio, J., *Perspective Drawing Handbook*, Tudor Press, 1964.
- Dubery, F., Willats, J., *Perspective and other drawing systems*, Van Nostrand Reinhold, 1972.
- Kingslake, R. *Optics in Photography*, SPIE Press, 1992.
- http://dpreview.com