The Light Field

Rays and throughput

Form factors

Light field representations

Hemispherical illumination

Illumination from uniform area light sources

Shadows: Blockers, umbras and penumbras

Radiosity

Rays and Throughput

Throughput = Measuring Rays

Define an infinitesimal beam as the set of rays intersecting two differential surface elements

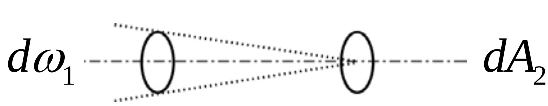
$$dT = \frac{dA_1 dA_2}{r^2}$$

Measure the number of rays in the beam
This quantity is called the *throughput*

Throughput = Measuring Rays

Parameterize rays wrt to source or receiver

$$dT = dA_1 d\omega_1 = dA_1 dA_2 / r^2$$



$$dT = dA_2 d\omega_2 = dA_1 dA_2 / r^2$$

Throughput = Measuring Rays

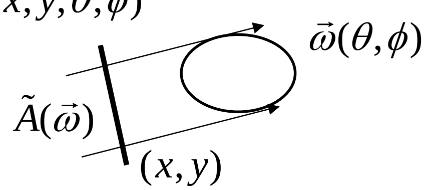
Tilting the surfaces parameterizes the rays wrt to the tilted surface

$$dT = \frac{\cos\theta \cos\theta'}{r^2} dA\, dA' = d\vec{\omega} \bullet d\vec{A}$$

Parameterizing Rays: $S^2 \times R^2$

Parameterize rays by $r(x, y, \theta, \phi)$

Projected area



Measuring the number that hit the shape

$$T = \int_{S^2} d\omega(\theta,\varphi) \int_{R^2} d\tilde{A}(x,y)$$

$$= \int_{S^2} \tilde{A}(\theta,\varphi)\, d\omega(\theta,\varphi)$$

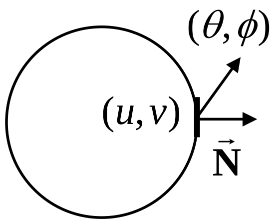
$$= 4\pi \bar{\tilde{A}}$$

Sphere:

$$T = 4\pi \bar{\tilde{A}} = 4\pi^2 r^2$$

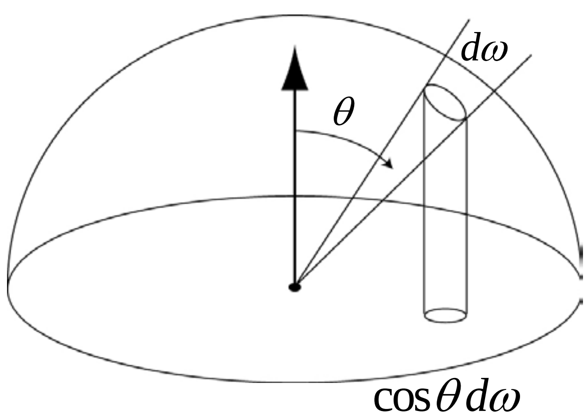
Parameterizing Rays: $M^2 \times S^2$

Parameterize rays by $r(u,v,\theta,\phi)$



$$T = \underbrace{\left[\int_{M^2} dA(u,v)\right]}_{S} \underbrace{\left[\int_{H^2(\vec{\mathbf{N}})} \cos\theta \, d\omega(\theta,\varphi)\right]}_{?}$$

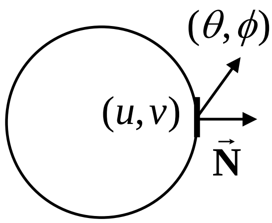
Projected Solid Angle



$$\int_{H^2} \cos\theta \, d\omega = \pi$$

Parameterizing Rays: $M^2 \times S^2$

Parameterize rays by $r(u,v,\theta,\phi)$



$$T = \underbrace{\left[\int_{M^2} dA(u,v)\right]}_{S} \underbrace{\left[\int_{H^2(\vec{\mathbf{N}})} \cos\theta \, d\omega(\theta,\varphi)\right]}_{\pi}$$

Sphere: $T = \pi S = 4\pi^2 r^2$

Crofton's Theorem: $4\pi \bar{\tilde{A}} = \pi S \Rightarrow \bar{\tilde{A}} = \dfrac{S}{4}$

Types of Throughput

1. Infinitesimal beam of rays

$$dT(dA,dA') = d\vec{\omega} \bullet d\vec{A} = \frac{\cos\theta\cos\theta'}{\left|x-x'\right|^2} dA(x)\, dA(x')$$

2. Differential-finite beam

$$T(dA,A')\,dA = \left[\int_{\Omega} \cos\theta \, d\omega(x')\right] dA(x) = \left[\int_{A'} \frac{\cos\theta\cos\theta'}{\left|x-x'\right|^2} dA(x')\right] dA(x)$$

3. Finite-finite beam

$$T(A,A') = \int_A \int_{\Omega} d\vec{\omega} \bullet d\vec{A} = \int_A \int_{A'} \frac{\cos\theta\cos\theta'}{\left|x-x'\right|^2} dA(x')\, dA(x)$$

Special case is one object with area *A*

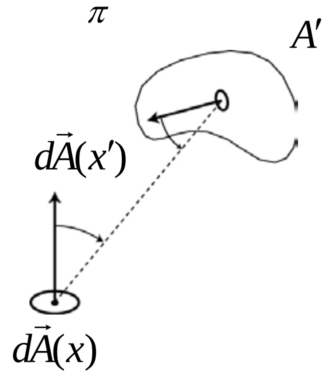
Form Factors

Differential Form Factor

Probability of a ray leaving *dA(x)* hitting *A′*

$$\Pr(A' \mid dA) = \frac{T(dA, A')dA}{T(dA)} = \frac{T(dA, A')dA}{\pi dA} = \frac{T(dA, A')}{\pi}$$

$$= \int_{A'} \frac{\cos\theta \cos\theta'}{\pi |x - x'|^2} dA(x')$$



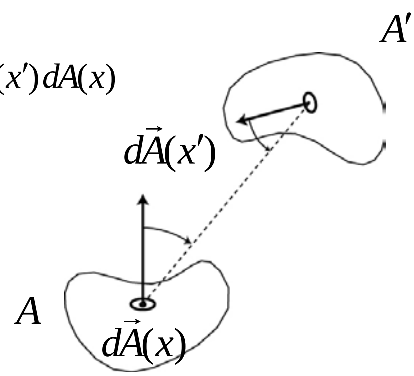
$$\int_{A'} G(x, x')\, dA(x') \equiv \int_{A'} \frac{\cos\theta \cos\theta'}{\pi |x - x'|^2} dA(x')$$

Form Factor

Probability of a ray leaving *A* hitting *A′*

$$\Pr(A' \mid A) = \frac{T(A', A)}{T(A)}$$

$$= \frac{1}{A} \int_A \int_{A'} \frac{\cos\theta \cos\theta'}{\pi |x - x'|^2} \, dA(x') \, dA(x)$$



Conservation of Radiance

Conservation of Throughput

- Throughput conversed during propagation
  - Number of rays conserved
  - Assuming no attenuation or scattering
- n^2 (index of refraction) times throughput invariant under the laws of geometric optics
  - Reflection at an interface
  - Refraction at an interface
    - Causes rays to bend (kink)
  - Continuously varying index of refraction
    - Causes rays to curve; mirages

Conservation of Radiance

Radiance is the ratio of two conserved quantities:

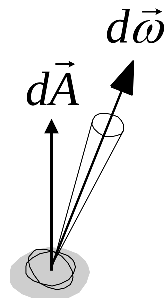
1. Power
2. Throughput

$$L(r) = \lim_{\Delta T \to 0} \frac{\Delta\Phi(\Delta T)}{\Delta T} = \frac{d\Phi}{dT}$$

$\therefore$ Radiance conserved

Surface Radiance

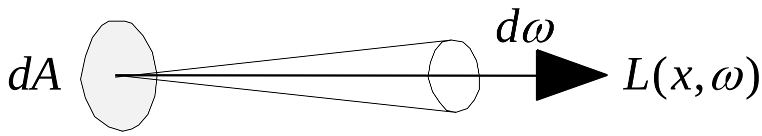
<u>Definition 1</u>: The surface *radiance* (*luminance*) is the intensity per unit projected area leaving a surface



$$L(x,\omega) \equiv \frac{d\Phi(x,\omega)}{d\vec{\omega} \bullet d\vec{A}}$$

Field Radiance

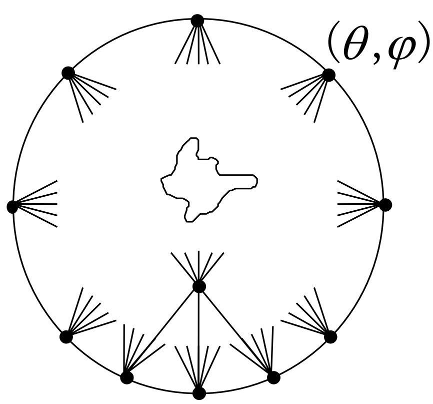
<u>Definition 1</u>: The field *radiance* (*luminance*) at a point in space in a given direction is the power per unit solid angle per unit area perpendicular to the direction



Light Field Representations

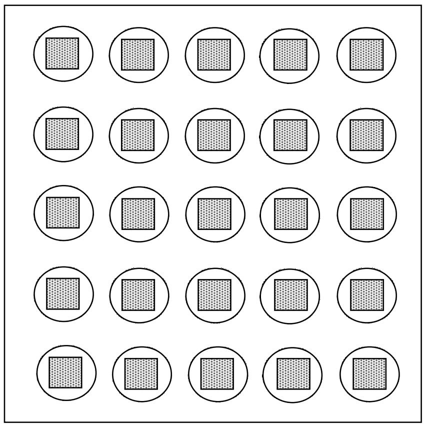
Spherical Light Field

$$L(x, y, \theta, \varphi)$$



Capture all the light leaving an object - like a hologram

Two-Plane Light Field



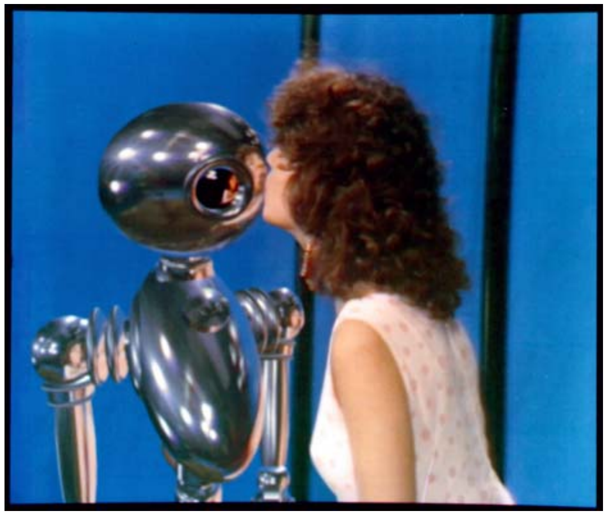
2D Array of Cameras **2D Array of Images**

$$L(u,v,s,t)$$

Environment Maps

Miller and Hoffman, 1984 $L(\theta,\varphi)$

Environment Maps



***Interface*, Chou and Williams (ca. 1985)**

The Sky

Plate 5-16. Fisheye view of clear sky at the South Pole. (Photographed by the author)

Plate 5-17. View of slightly hazy sky in Wisconsin. (Photographed by the author)

From Greenler, Rainbows, halos and glories

Irradiance from a Hemisphere

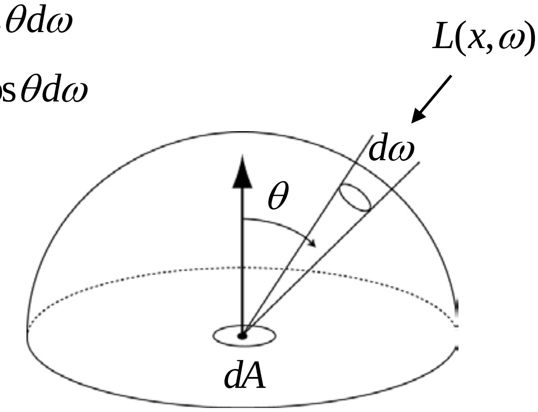
Irradiance from a Hemisphere

$$d\Phi(x) = L(x,\omega)\cos\theta \, d\omega \, dA = dE \, dA$$

$$dE(x) = L(x,\omega)\cos\theta \, d\omega$$

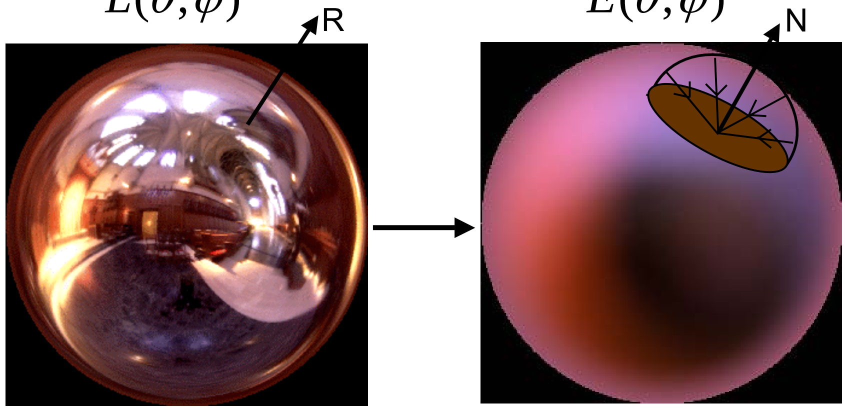
$$E(x) = \int_{H^2} L(x,\omega)\cos\theta \, d\omega$$

Light meter



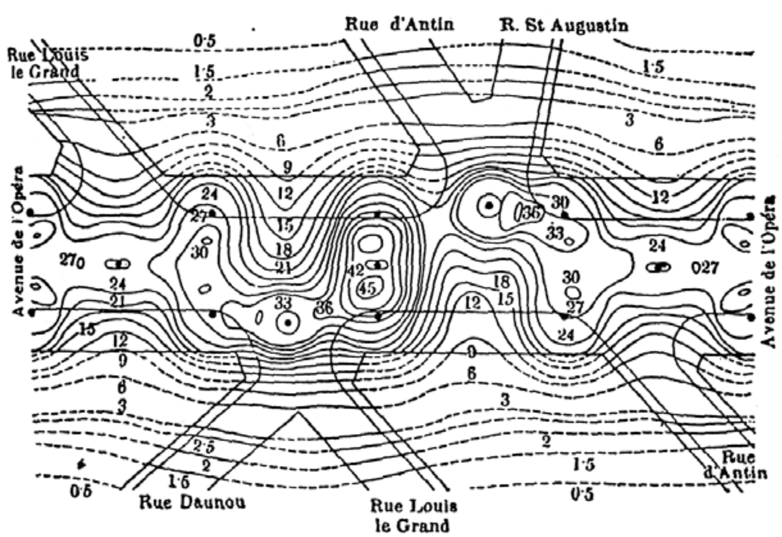
Irradiance Environment Maps

$L(\theta,\varphi)$ $E(\theta,\varphi)$



Radiance Environment Map **Irradiance Environment Map**

Irradiance Map or Light Map



Isolux contours

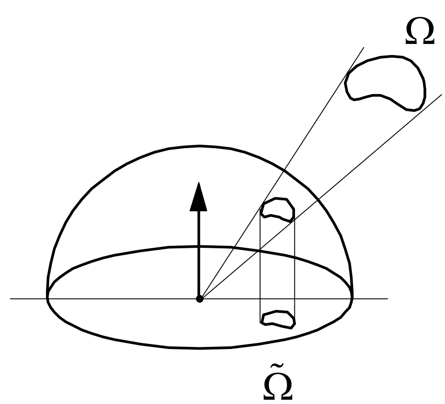
Uniform Area Sources

Irradiance from an Area Source

$$E(x) = \int_{H^2} L\cos\theta d\omega$$

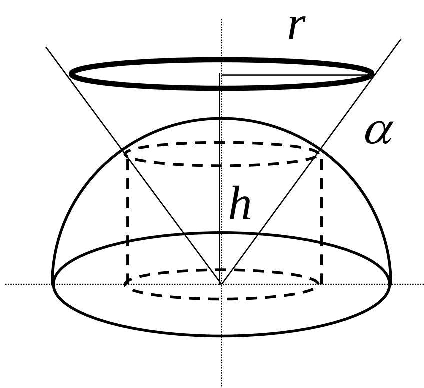
$$= L\int_{\Omega} \cos\theta d\omega$$

$$= L\tilde{\Omega}$$



Disk Source

Geometric Derivation



$$\tilde{\Omega} = \pi \sin^2 \alpha$$

Algebraic Derivation

$$\tilde{\Omega} = \int_1^{\cos\alpha} \int_0^{2\pi} \cos\theta \, d\phi \, d\cos\theta$$

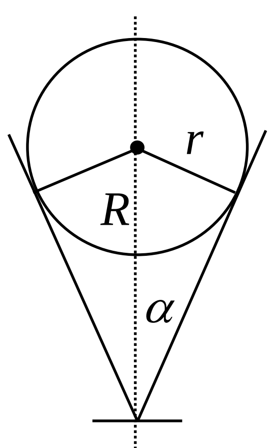
$$= 2\pi L \left. \frac{\cos^2\theta}{2} \right|_1^{\cos\alpha}$$

$$= L\pi \sin^2 \alpha$$

$$= L\pi \frac{r^2}{r^2 + h^2}$$

Spherical Source

Geometric Derivation



$$\tilde{\Omega} = \pi \sin^2 \alpha$$

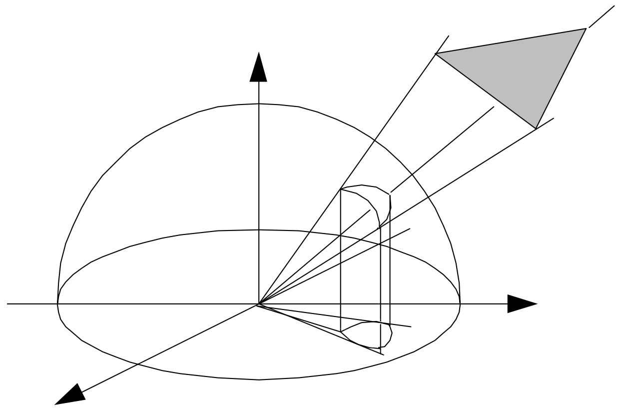
Algebraic Derivation

$$\tilde{\Omega} = \int \cos\theta \, d\omega$$

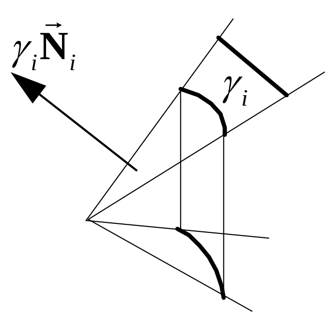
$$= \pi \sin^2 \alpha$$

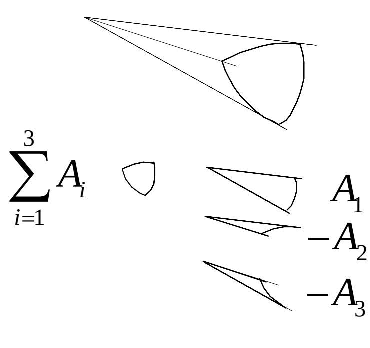
$$= \pi \frac{r^2}{R^2}$$

Polygonal Source



Lambert's Formula

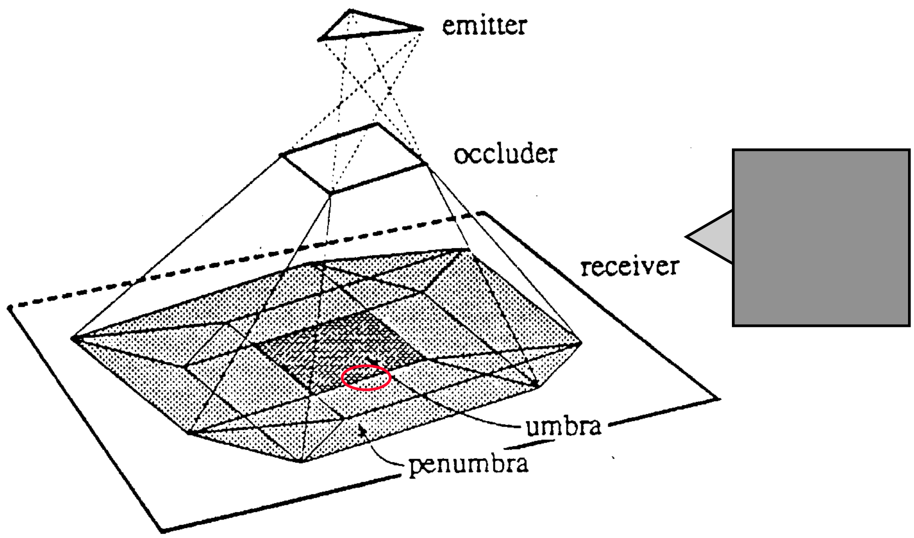




$$A_i = \gamma_i \vec{\mathbf{N}}_i \bullet \vec{\mathbf{N}}$$

$$\sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \gamma_i \vec{\mathbf{N}}_i \bullet \vec{\mathbf{N}}$$

Penumbras and Umbras



CS348B Lecture 5 Pat Hanrahan, Spring 2002

Radiosity

Radiosity and Luminosity

<u>Definition</u>: The *radiosity* (*luminosity*) is the energy per unit area leaving a surface.

$$M(x) \equiv \frac{d\Phi_o}{dA}$$

$$\left[\frac{W}{m^2}\right] \left[\frac{lm}{m^2} = lux\right]$$

This is officially referred to as the *radiant* (*luminous*) *exitance.*

Uniform Diffuse Emitter

$$d\Phi_o(x) = L_o(x,\omega)\cos\theta \, d\omega \, dA = M \, dA$$

$$dM(x) = L_o(x,\omega)\cos\theta \, d\omega$$

$$M(x) = \int_{H^2} L_o(x,\omega)\cos\theta \, d\omega$$

$$M = \int L \cos\theta \, d\omega = \pi L$$

$$L = \frac{M}{\pi}$$

The Sun

Solar constant (normal incidence at zenith)

Irradiance **1353 W/m²**

Illuminance **127,500 lm/m² = 127.5 kilolux**

Solar angle

α **= .25 degrees = .004 radians (half angle)**

$\Omega = \pi \sin^2 \alpha = 6 \times 10^{-5}$ **steradians**

Radiance

$$L = \frac{E}{\tilde{\Omega}} = \frac{1.353 \times 10^3 \, W / m^2}{6 \times 10^{-5} \, sr} = 2.25 \times 10^7 \, \frac{\mathrm{W}}{m^2 \cdot sr}$$