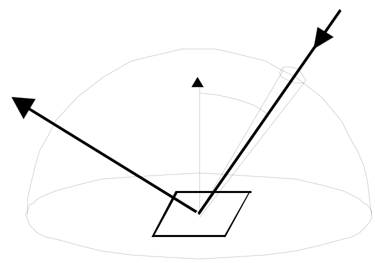
# Illumination Models

To evaluate the reflection equation the illumination must be specified or computed.

$$L_r(x,w_r) = \int_{H^2} f_r(x,w_i \rightarrow w_r) L_i(x,w_i) \cos q_i' \, dw_i$$

- Direct (*local*) illumination
  - Light directly from light sources
  - No shadows
- Indirect (*global*) illumination
  - Shadows due to blocking light
  - Interreflections
  - Complete accounting of light energy



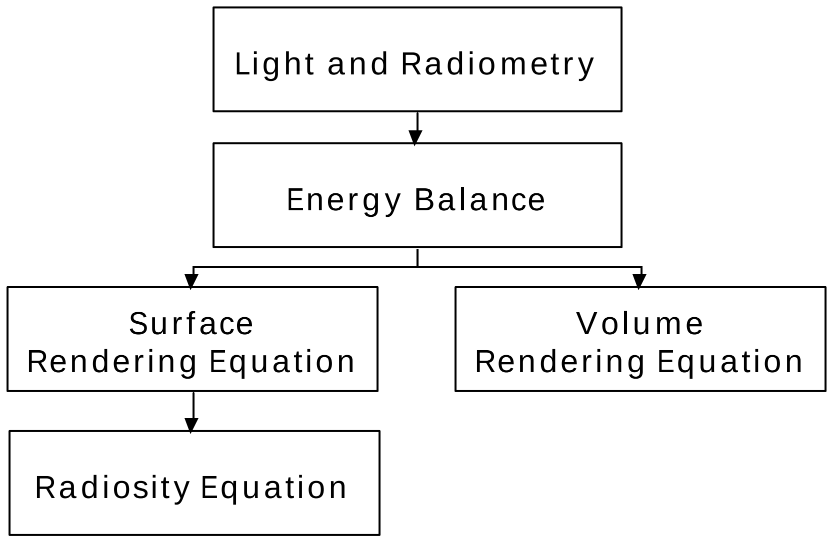


# To The Rendering Equation

## Questions

1. How is light measured?
2. How is the spatial distribution of light energy described?
3. How is reflection from a surface characterized?
4. What are the conditions for equilibrium flow of light in an environment?

# The Grand Scheme






# Balance Equation

Accountability

[*outgoing*] - [*incoming*] = [*emitted*] - [*absorbed*]

- Macro level

  *The total light energy put into the system must equal the energy leaving the system (usually, via heat).*

$$\Phi_o - \Phi_i = \Phi_e - \Phi_a$$

- Micro level

  *The energy flowing into a small region of phase space must equal the energy flowing out.*

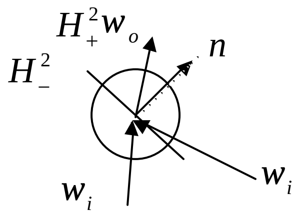
$$B(x) - E(x) = B_e - E_a$$

$$L_o(x,w) - L_i(x,w) = L_e(x,w) - L_a(x,w)$$

## Surface Balance Equation

[*outgoing*] = [*emitted*] + [*reflected*] + [*transmitted*]

$$L_o(x,w_o) = L_e(x,w_o) + L_r(x,w_o) + L_t(x,w_o)$$




$$= L_e(x,w_o)$$
$$+ \int_{H^2_+} f_r(x,w_i \rightarrow w_o) L_i(x,w_i) \cos q_i' dw_i$$
$$+ \int_{H^2_-} f_t(x,w_i \rightarrow w_o) L_i(x,w_i) \cos q_i' dw_i$$

$H^2_-(x) \quad w_i \bullet n(x) > 0$

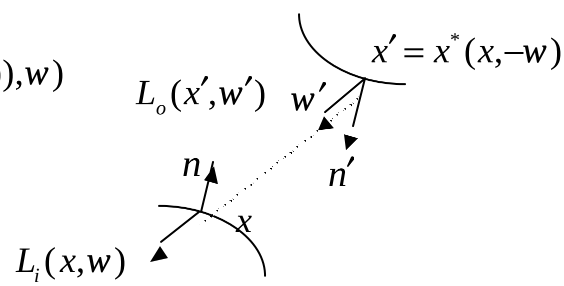
$H^2_+(x) \quad w_i \bullet n(x) < 0$



## The Rendering Equation

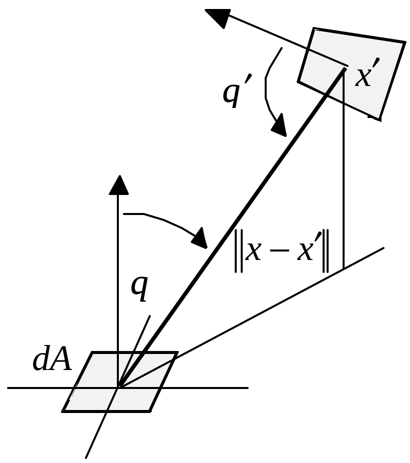
Couple balance equations

$$L_i(x,w) = L_o(x^*(x,-w)),w)$$




$$L_o(x,w_o) = L_e(x,w_o) + \int_{H^2_+} f_r(x,w_i \rightarrow w_o) L_o(x^*(x,-w_i),w_i) \cos q_i \, dw_i$$

## Two Point Geometry




$$dw = \frac{\cos q'}{\left\| x - x' \right\|^2} dA'$$

$$\cos q \; dw = \frac{\cos q \, \cos q'}{\left\| x - x' \right\|^2} dA'$$



## The Rendering Equation

$$L_o(x,w_o) = L_e(x,w_o) + \int_{M^2} f_r(x, w_i(x - x') \to w_o) G(x,x') L_o(x', w'_o(x - x'))\, dA'$$

Integrate over
All surfaces

Geometry term

$$G(x,x') = \frac{\cos q_i \, \cos q'_o}{\left\| x - x' \right\|^2} V(x,x')$$

$$V(x,x') = \begin{cases} 1 & \text{visible} \\ 0 & \text{not visible} \end{cases}$$

Visibility term

## The Radiosity Equation

Assume diffuse reflection

1. $f_r(x, w_i \rightarrow w_o) = f_r(x) \quad \Rightarrow \quad r(x) = p f_r(x)$
2. $L_o(x, w) = B(x)/p$

$$B(x) = B_e(x) + r(x) E(x)$$

$$B(x) = B_e(x) + r(x) \int_{M^2} F(x, x') B(x')\, dA'$$

$$F(x, x') = \frac{G(x, x')}{p}$$

Form factor: The percentage of light leaving ***dA'*** that arrives at ***dA***



## Integral Equations

Integral equations of the 1st kind

$$f(x) = \int k(x, x') g(x')\, dx'$$

Integral equations of the 2nd kind

$$f(x) = g(x) + \int k(x, x') f(x')\, dx'$$

## Linear Operators

*Linear operators act on functions like matrices act on vectors*

$$h(x) = (K \circ f)(x) \equiv \int k(x, x') f(x') dx'$$

They are linear in that

$$(K \circ (af + bg)) = aK \circ f + bK \circ g$$

Other types of linear operators, e.g. derivatives, integro-differential operators



## Formal Solution of Integral Equations

Integral equation

$$B = B_e + K \circ B$$

$$(I - K) \circ B = B_e$$

Formal solution

$$B = (I - K)^{-1} \circ B_e$$

Neumann series

$$\frac{1}{1-x} = 1 + x + x^2 + \ldots$$

$$\frac{1}{I-K} = I + K + K^2 + \ldots$$

## Formal Solution of Integral Equations

Successive Approximation

$$\frac{1}{I-K}B_e = B_e + KB_e + K^2B_e + \dots$$
$$= (B_e + K(B_e + K(B_e + \dots$$

$$B_1 = B_e$$
$$B_2 = B_e + KB_1$$
$$B_3 = B_e + KB_2$$
$$\dots$$
$$B_n = B_e + KB_{n-1}$$



## Formal Solution

Formal solution to the rendering equation

$$L(x,w) = \sum_{k=0}^{\infty} \int_{M^2} \int_{M^2} \dots \int_{M^2} K(x_o, x_1, x_2) \cdots K(x_k, x, w) L_e(x_0, x_1)\, dA_0 dA_1 \dots dA_k$$

Sum over all *paths* of length k

CS348B Lecture 12 Pat Hanrahan, Spring 2000

## Two Types of Operators

1. Transport operator

$$L(x,w) = T \circ L(x',w') = L(x^*(x,-w),w)$$

2. Scattering operator

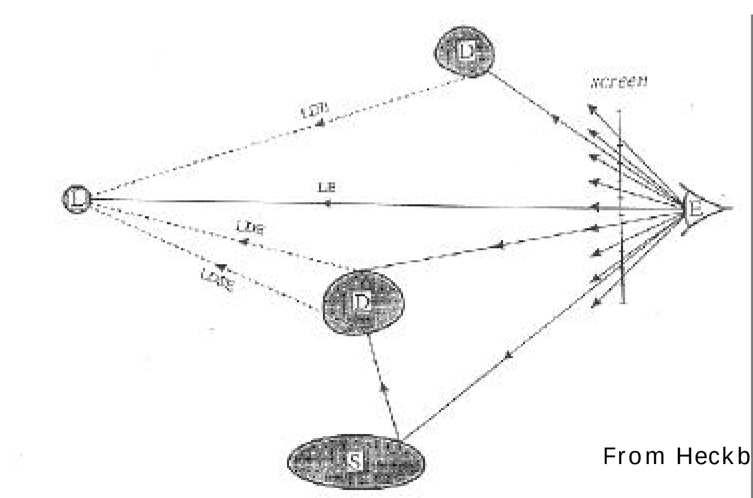
$$L(x,w_o) = S \circ L(x,w_i) = \int_{H_+^2} f_r(x,w_i \rightarrow w_o) L_i(x,w_i) \cos q_i \, dw_i$$

Rendering equation

$$(I - K) \circ L = (I - S \circ T) \circ L = L_e$$



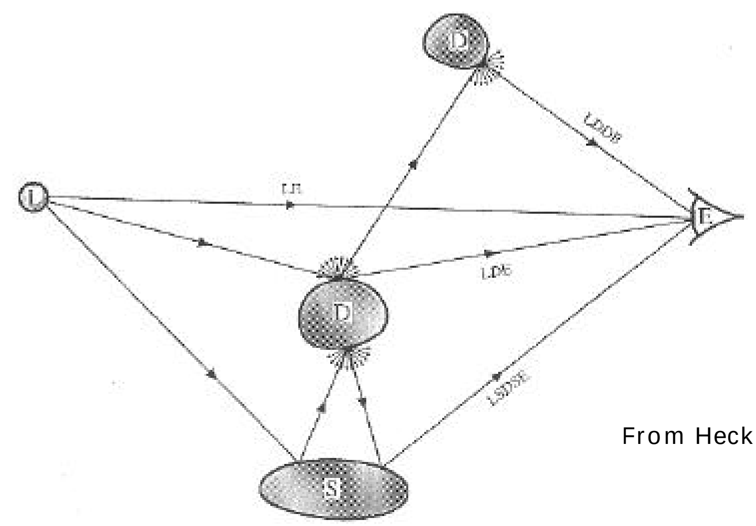
## Classic Ray Tracing




From Heckbert

Forward (from eye): E S* (D|G) L

## Photon Paths




From Heckbert



## How to Solve It?

Finite element methods

- Classic radiosity
  - Mesh surfaces
  - Piecewise constant basis functions
  - Solve matrix equation
- Not practical for rendering equation

Monte Carlo methods

- Distributed ray tracing
  - Randomly traces ray from the eye
- Path tracing
- Bidirectional ray tracing
- Photon tracing