# Optics II: practical photographic lenses

CS 178, Spring 2013

Begun 4/11/13, finished 4/16/13.

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Stanford University

# Outline

- why study lenses?
- thin lenses
  - graphical constructions, algebraic formulae
- thick lenses
  - center of perspective, 3D perspective transformations
- depth of field

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- aberrations & distortion
- vignetting, glare, and other lens artifacts
- diffraction and lens quality
- special lenses
  - telephoto, zoom

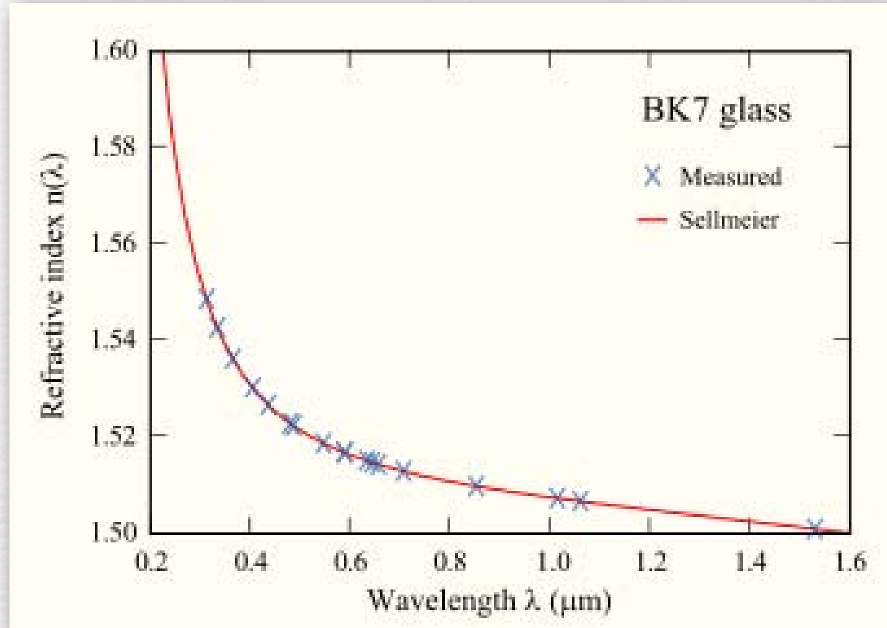
# Lens aberrations

- chromatic aberrations
- Seidel aberrations, a.k.a. 3rd order aberrations
  - arise because we use spherical lenses instead of hyperbolic
  - can be modeled by adding 3rd order terms to Taylor series

$$\sin\phi \approx \phi \left( -\frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \frac{\phi^7}{7!} + \ldots \right)$$

  - oblique aberrations
  - field curvature
  - distortion

# Dispersion

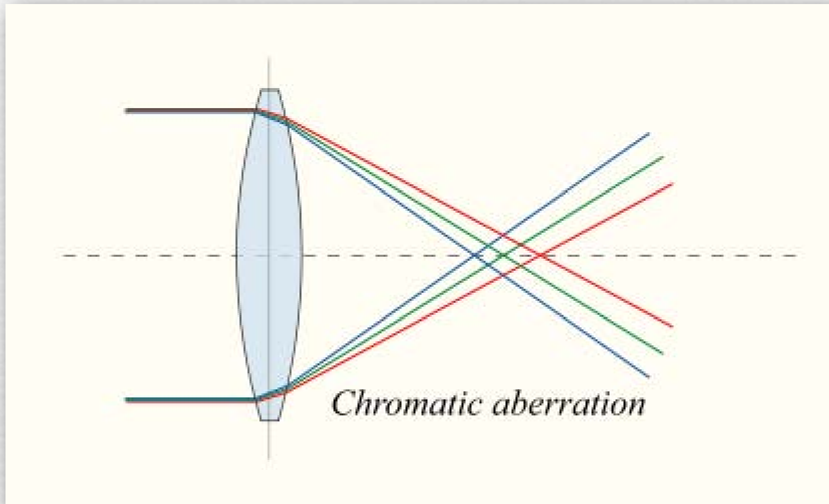


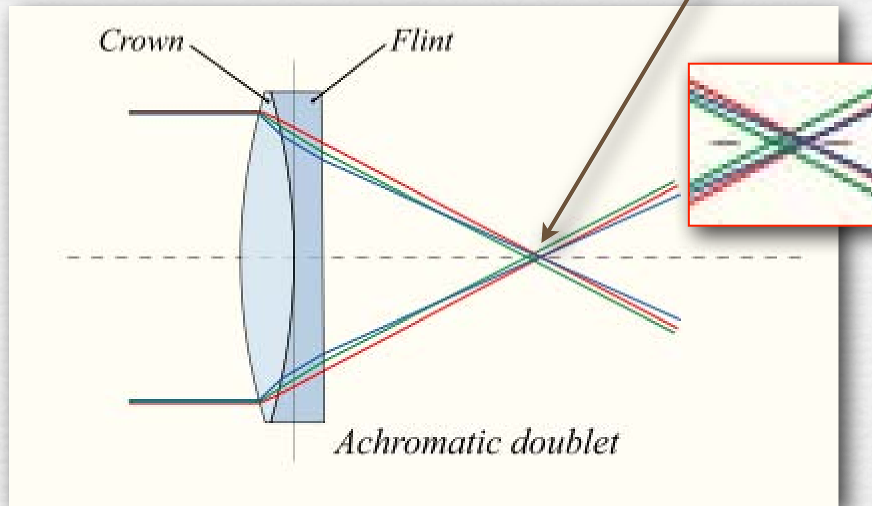

(wikipedia)

- index of refraction varies with wavelength
  - higher dispersion means more variation
  - amount of variation depends on material
  - index is typically higher for blue than red
  - so blue light bends more

# Chromatic aberration

red and blue have the same focal length

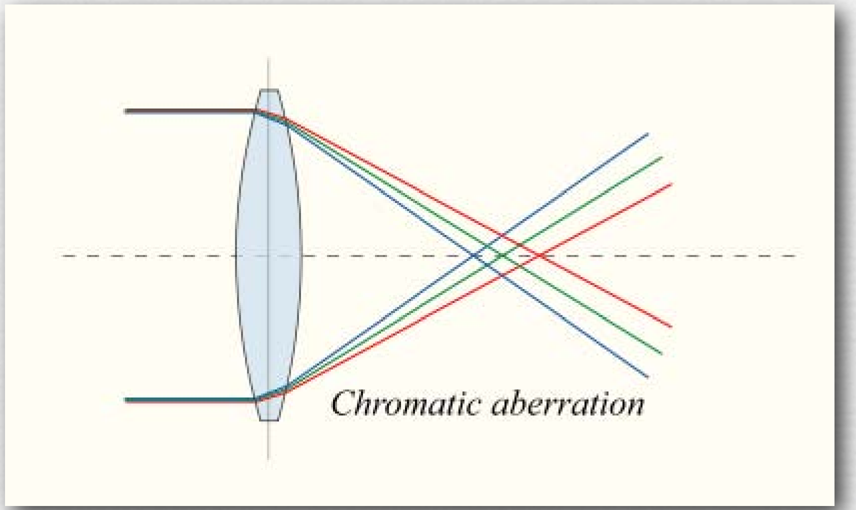


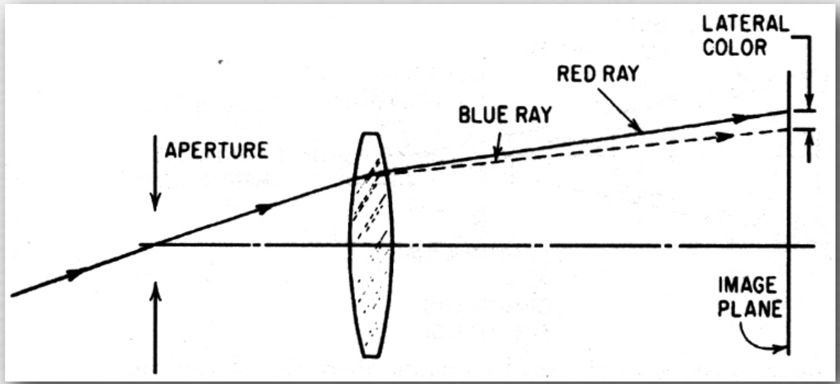




(wikipedia)

- dispersion causes focal length to vary with wavelength
  - for convex lens, blue focal length is shorter
- correct using *achromatic doublet*
  - strong positive lens + weak negative lens = weak positive compound lens
  - by adjusting dispersions, can correct at two wavelengths

# The chromatic aberrations




(Smith)

- *longitudinal (axial) chromatic aberration*
  - different colors focus at different depths
  - appears everywhere in the image
- *lateral (transverse) chromatic aberration*
  - if blue image is closer to lens, it will also be smaller
  - only appears at edges of images, not in the center
- can reduce longitudinal by closing down the aperture

# Examples

- correctable in software
- not easily correctable

(wikipedia)

lateral

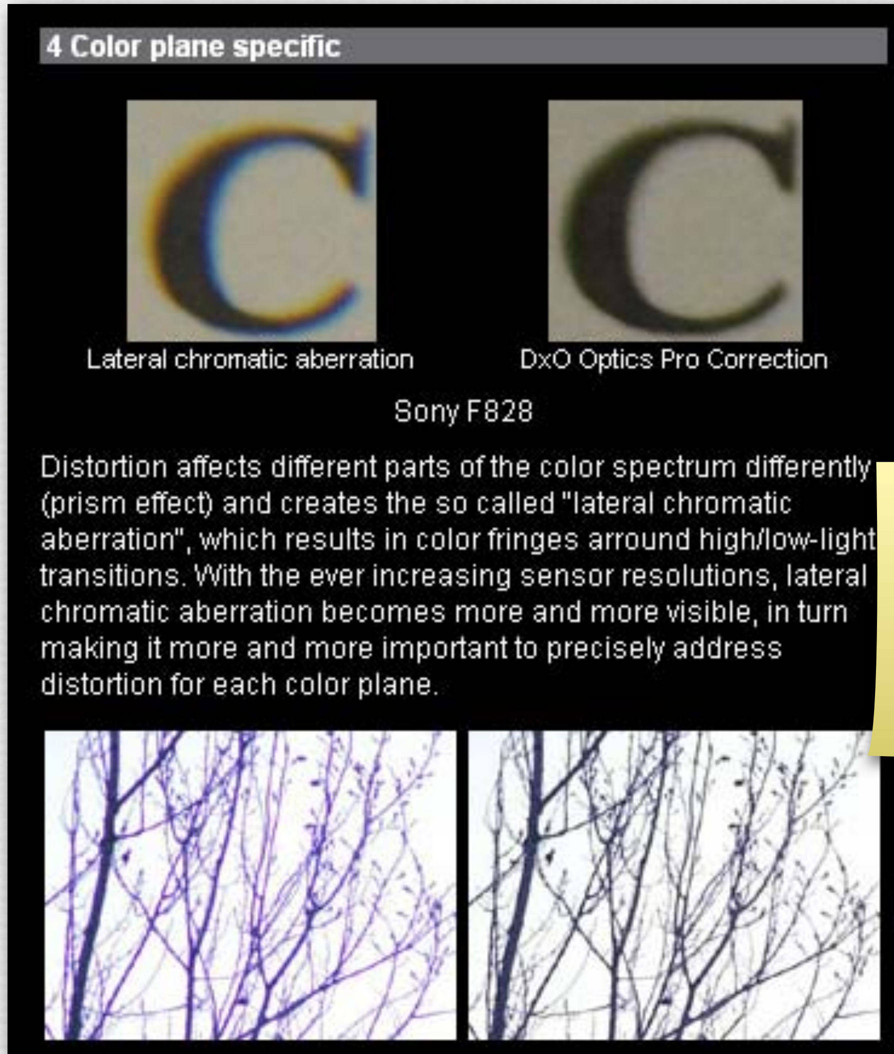
(cropped from edge of image)

(toothwalker.org)

longitudinal

✦ other possible causes of color fringing
  - demosiacing algorithm
  - per-pixel microlenses
  - lens flare

# Software correction of lateral chromatic aberration



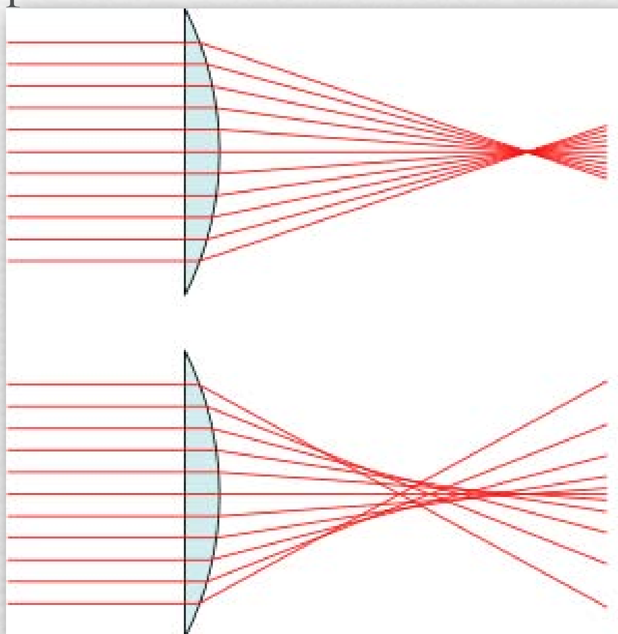

- Panasonic GX1 corrects for chromatic aberration in the camera (or in Adobe Camera Raw)
  - need focal length of lens, and focus setting

A student asked whether in a 3-CCD color camera (which we'll cover later in the course), one could optimize the position of each of the 3 sensors to minimize longitudinal chromatic aberration. Yes. The student went on to ask whether one could use piezos to move the 3 sensors axially as the lens zooms or focuses, hence presumably changing precisely where the red, green, and blue components would focus. I have never heard of such an adjustment, but I'll try to find out.

Q. Why don't humans see chromatic aberration?

# Spherical aberration

hyperbolic lens



spherical lens (wikipedia)

- focus varies with ray height (distance from optical axis)
- can reduce by stopping down the aperture
- can correct using an aspherical lens
- can correct for this and chromatic aberration
  by combining with a concave lens of different properties

# Examples

(Canon)

sharp

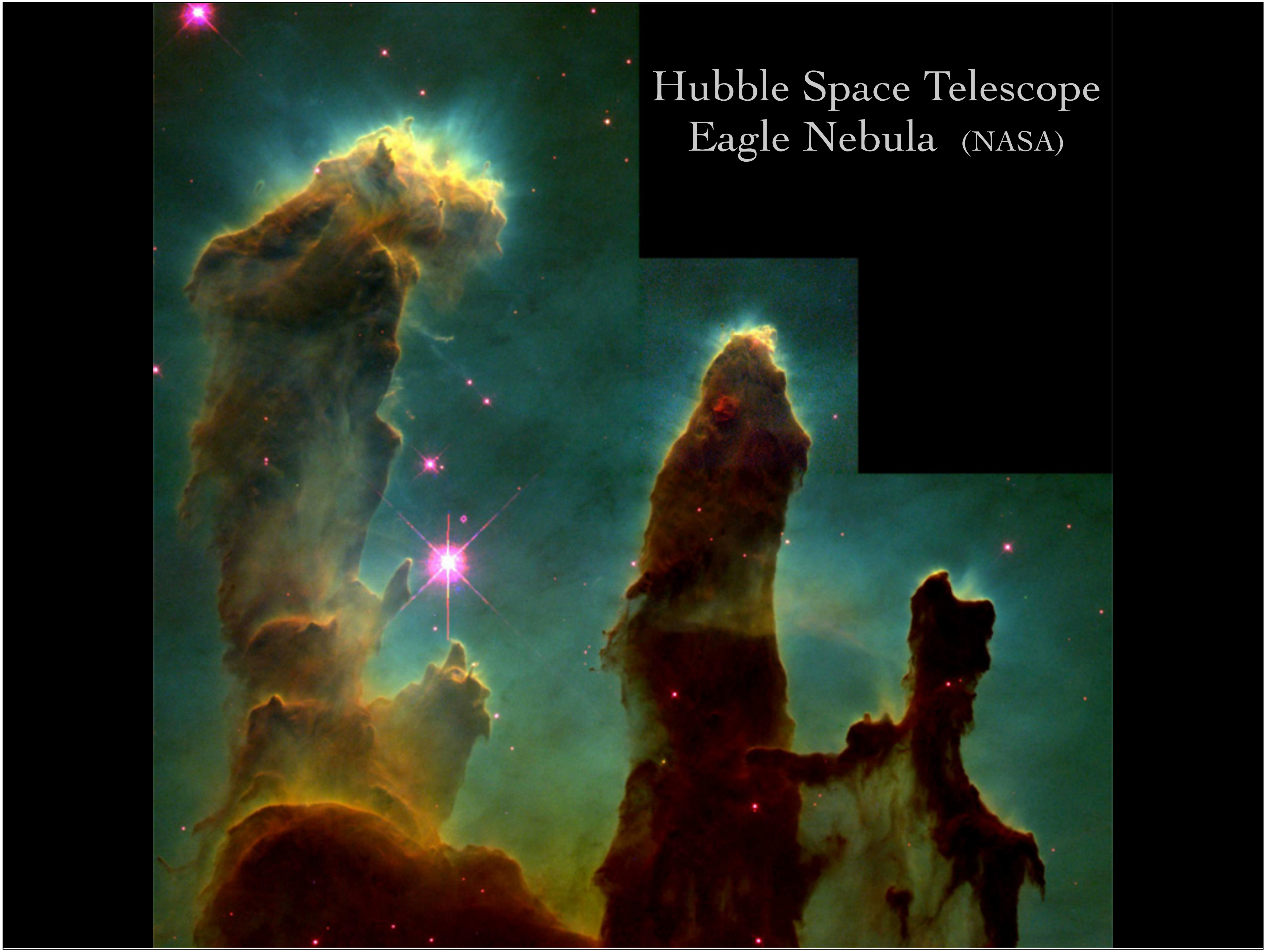
soft focus

Canon 135mm f/2.8 soft focus lens

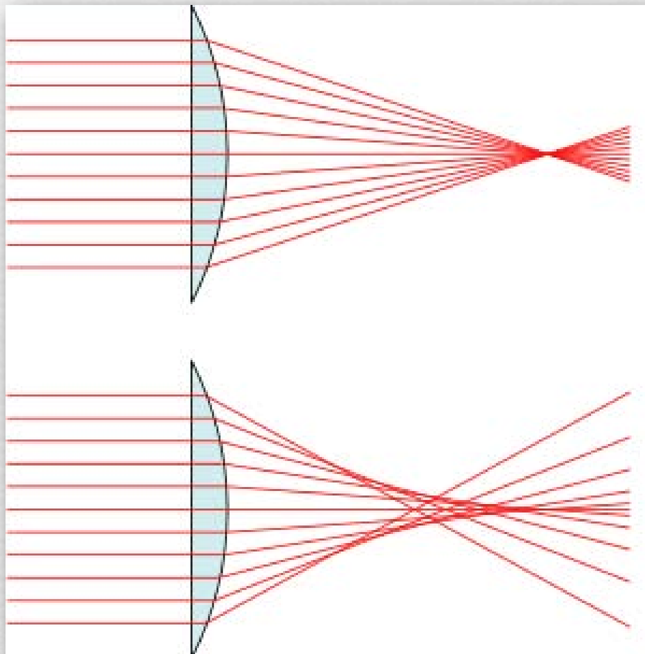
# Hubble telescope

before correction

after correction


Hubble Space Telescope
Eagle Nebula (NASA)

# Focus shift



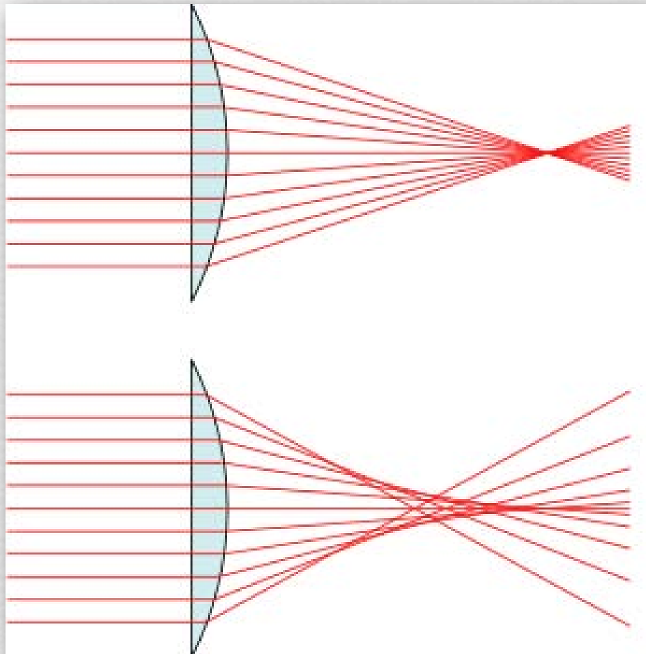
(wikipedia)

(diglloyd.com)

focused at f/1.2

- Canon 50mm f/1.2 L

# Focus shift



(wikipedia)

(diglloyd.com)

shot at f/1.8

- Canon 50mm f/1.2 L
- narrowing the aperture pushed the focus deeper

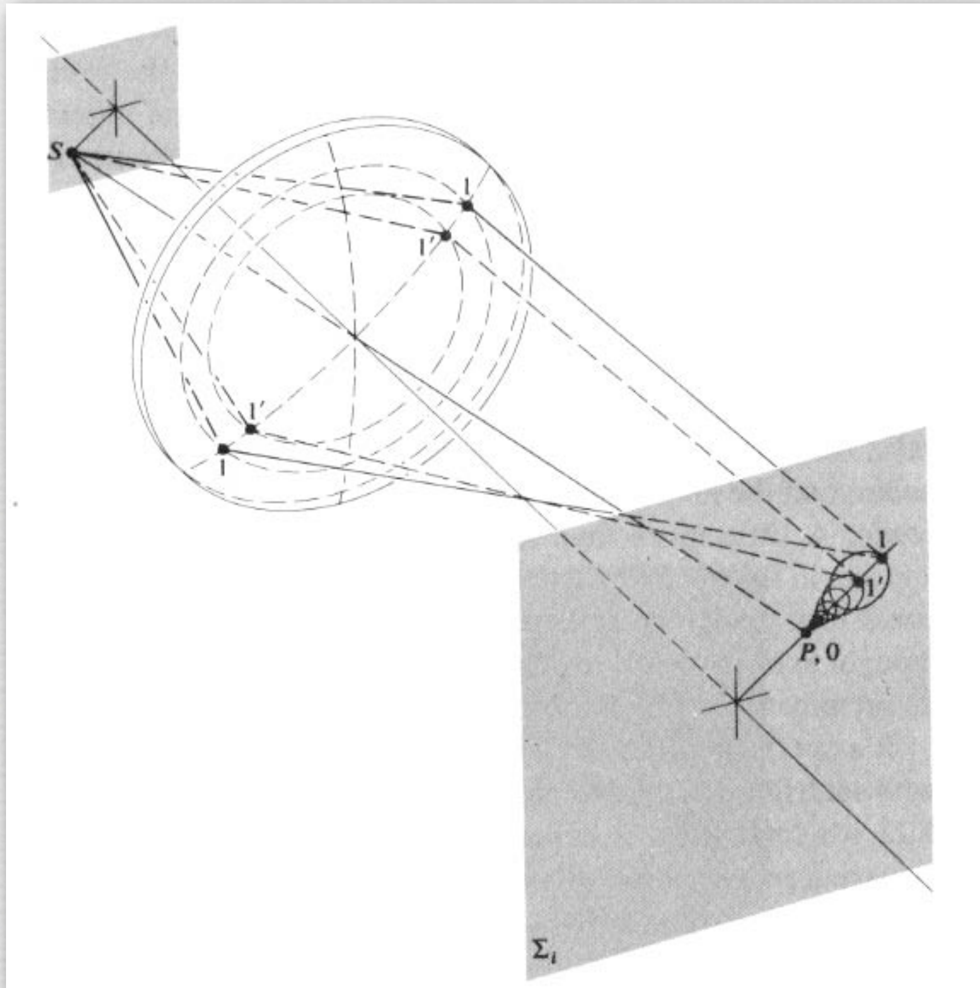
# Aberrations and their properties

- lateral chromatic aberrations do not appear in center of field
  - they get worse with increasing distance from the optical axis
  - cannot reduce by closing down the aperture

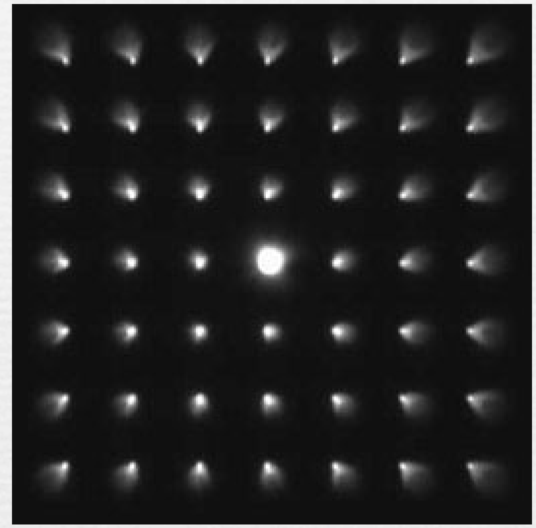
- longitudinal chromatic & spherical aberrations occur everywhere in the field of view
  - on and off the optical axis
  - can reduce by closing down the aperture

- oblique aberrations do not appear in center of field
  - they get worse with increasing distance from the optical axis
  - can reduce by closing down the aperture
  - coma and astigmatism

# Coma



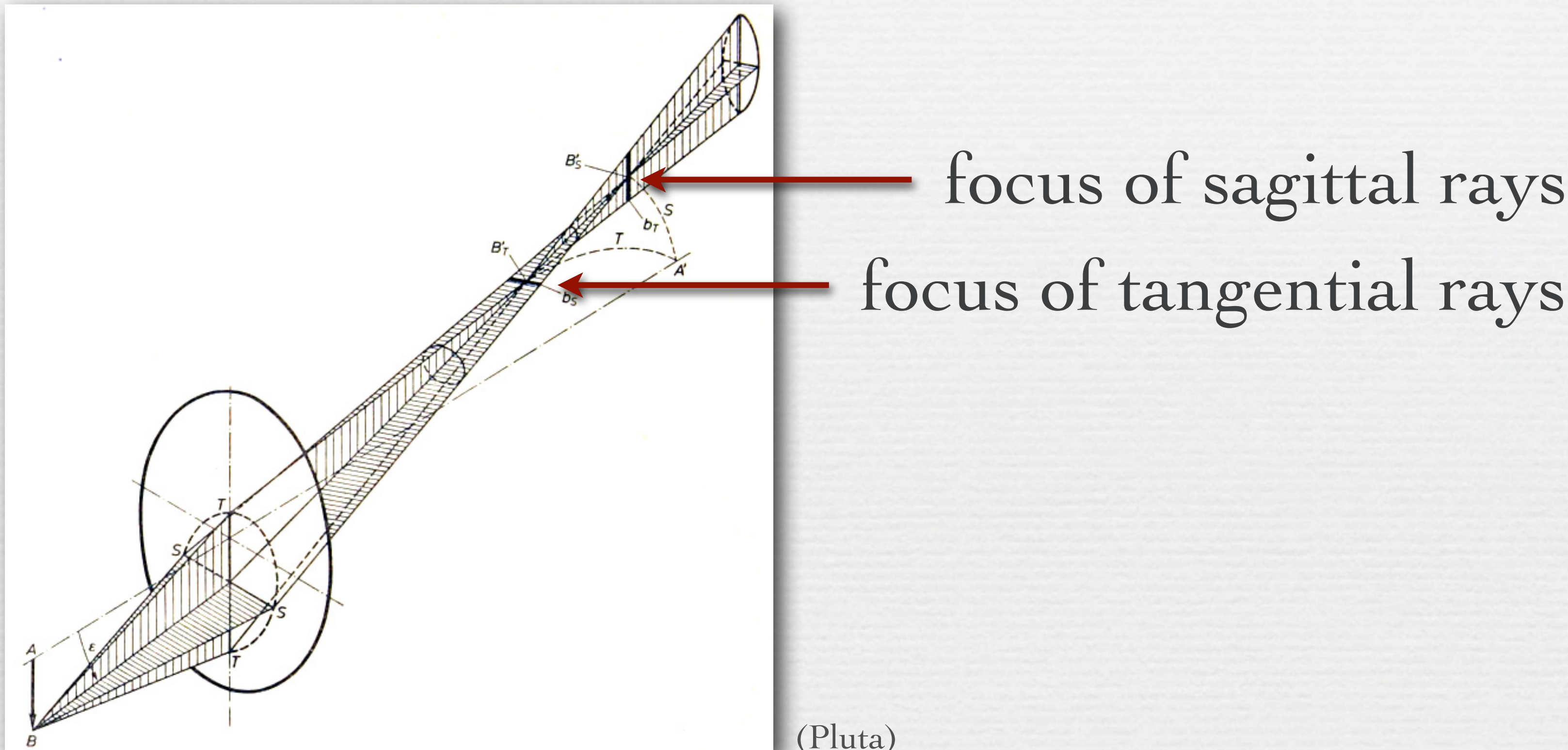

(Hecht)



(ryokosha.com)

- magnification varies with ray height (distance from optical axis)

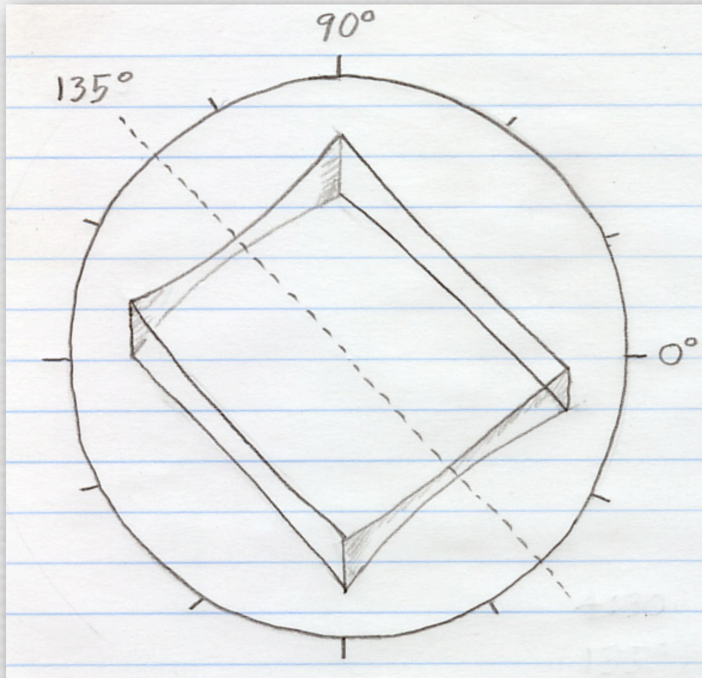
# Astigmatism




(Pluta)

- tangential and sagittal rays focus at different depths
- my full eyeglass prescription
  - right: -0.75 -1.00 axis 135, left: -1.00 -0.75 axis 180

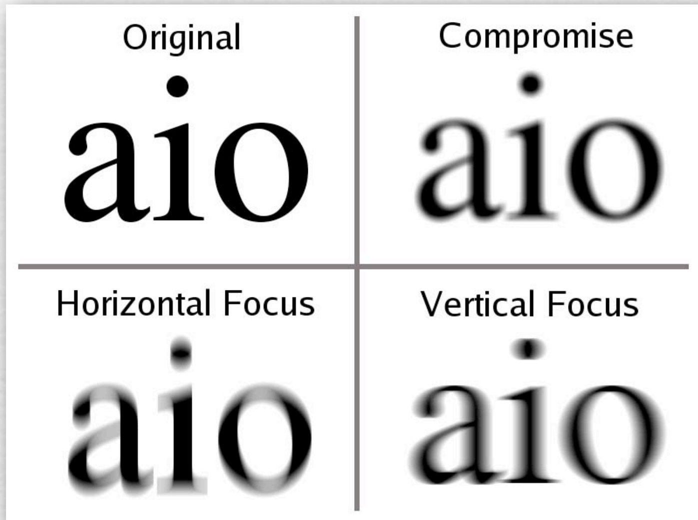
# Correcting astigmatism using a cylindrical lens (contents of whiteboard)




- for myopia + astigmatism, one needs a spherical lens + cylindrical lens, i.e. a lens with different radii of curvature in two perpendicular directions
  - in my right eye, first direction has focal length -1 /0.75 = -1.33 meters, and second direction has focal length -1 / 1.00 = -1.00 meters
- lens is then rotated around the optical axis before mounting in frame
  - in my case extrusion axis of second curvature is 135° (10:30 - 4:30 on the clock)

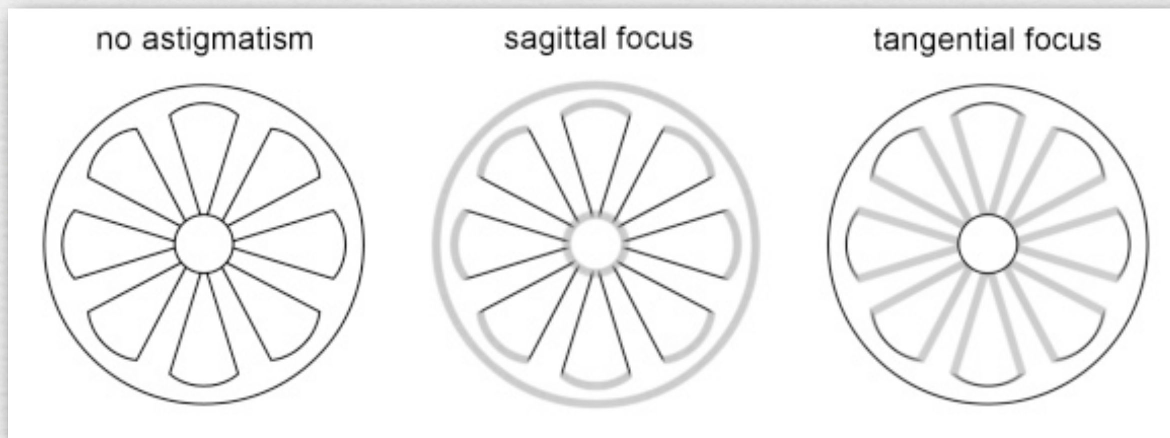
# Two kinds of astigmatism

(Wikipedia)



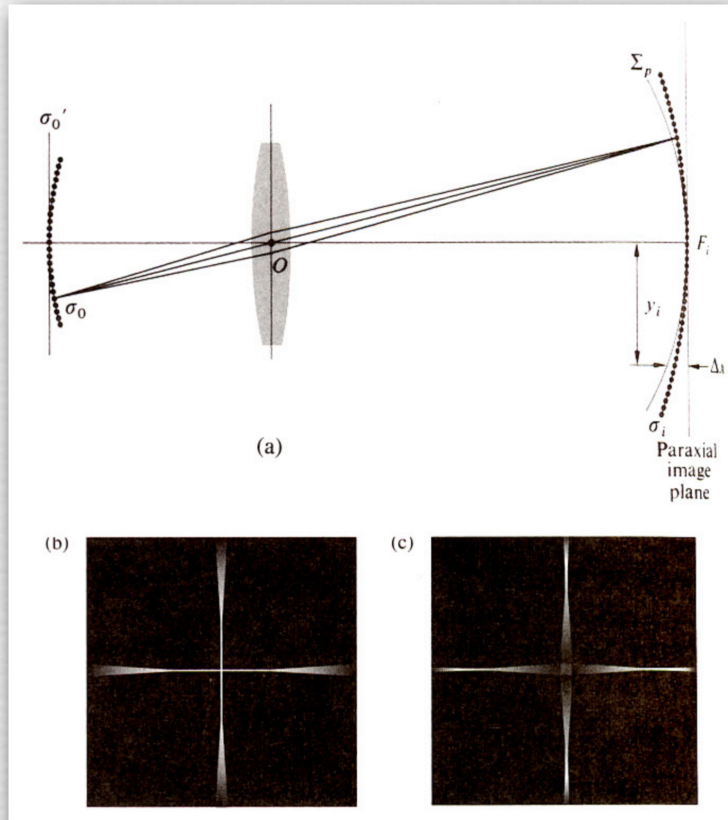

ophthalmic astigmatism
(due to oblong eye)

(http://toothwalker.org/optics/astigmatism.html)




third-order astigmatism
(even in rotationally symmetric photographic lenses)

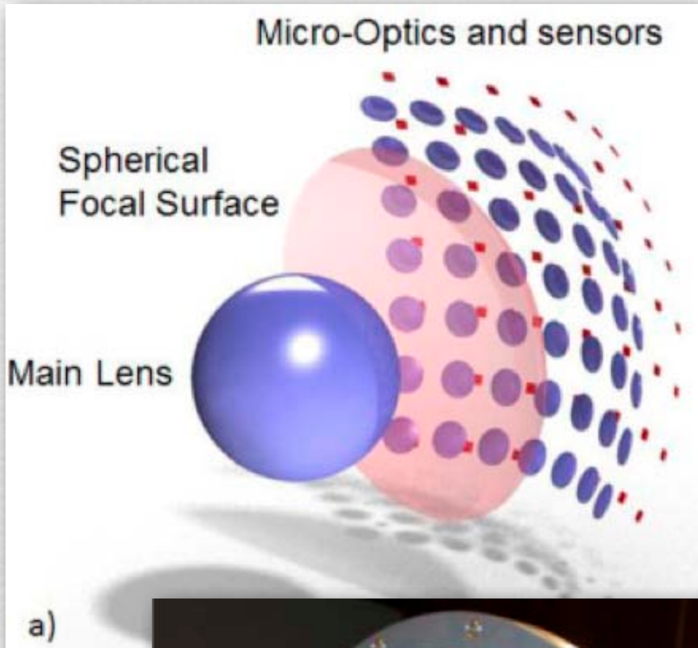
# Field curvature

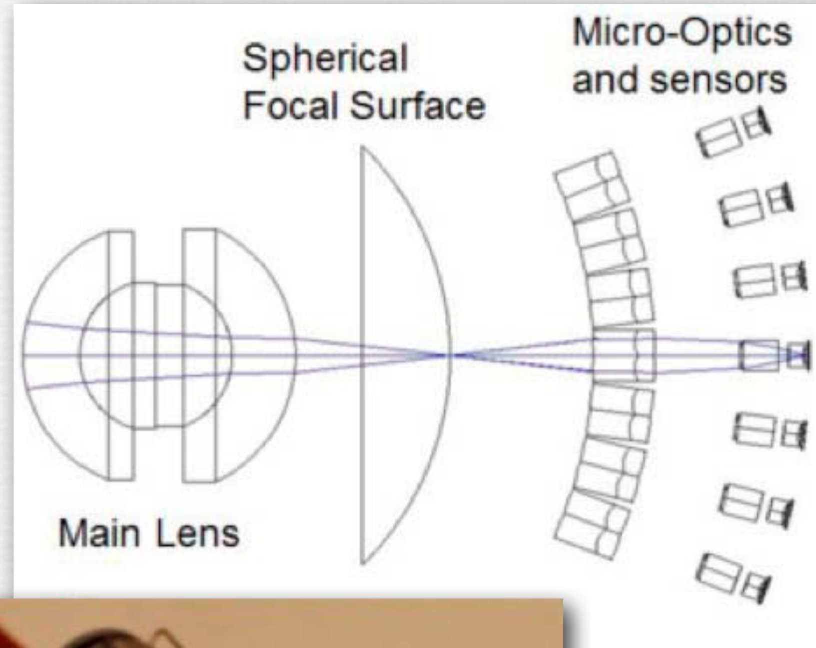


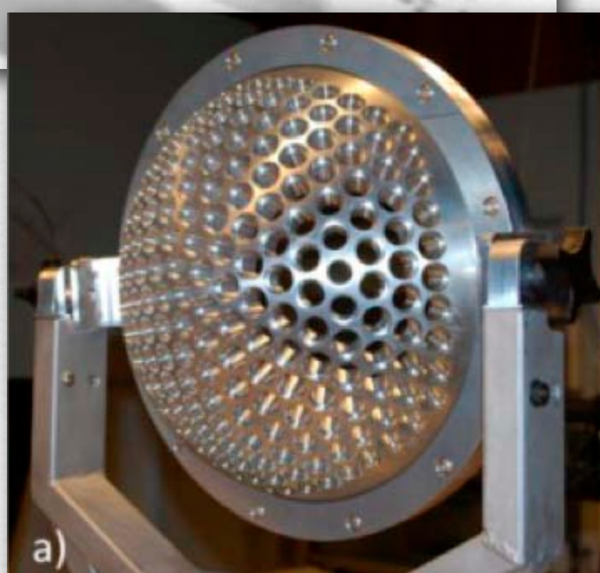

(Hecht)

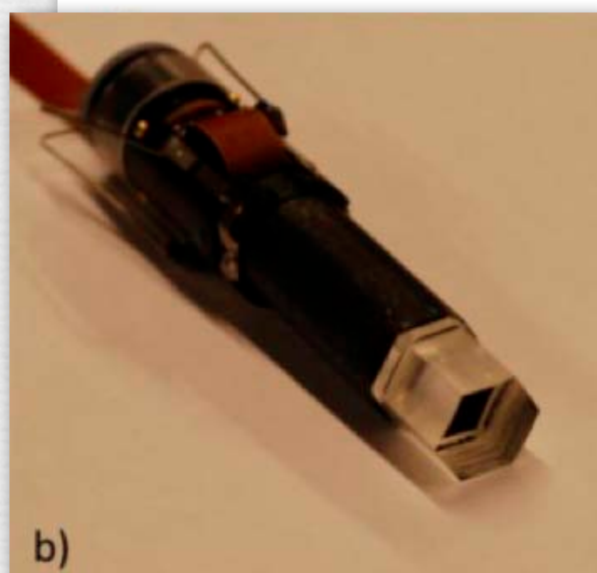
- spherical lenses focus a curved surface in object space onto a curved surface in image space
- so a plane in object space cannot be everywhere in focus when imaged by a planar sensor

# A spherical focus surface camera



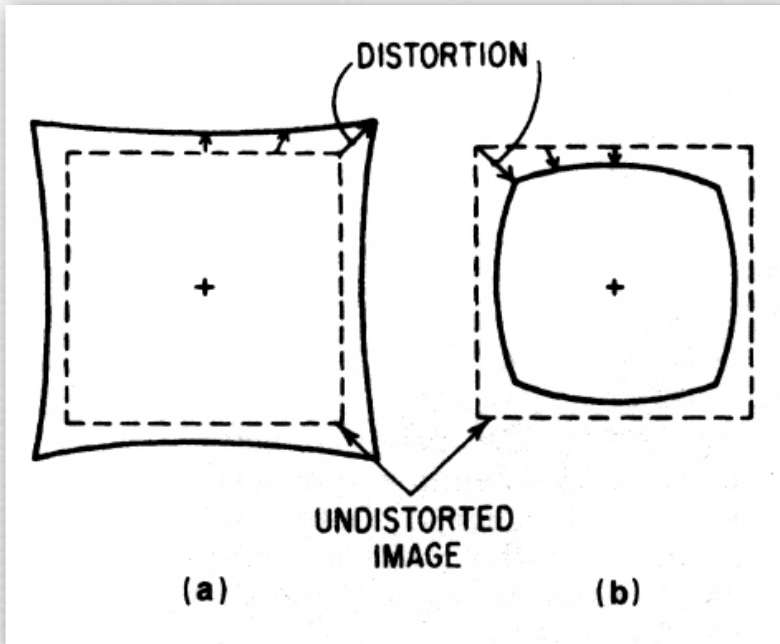




[Son 2011]

# Distortion

- correctable in software

(Smith)



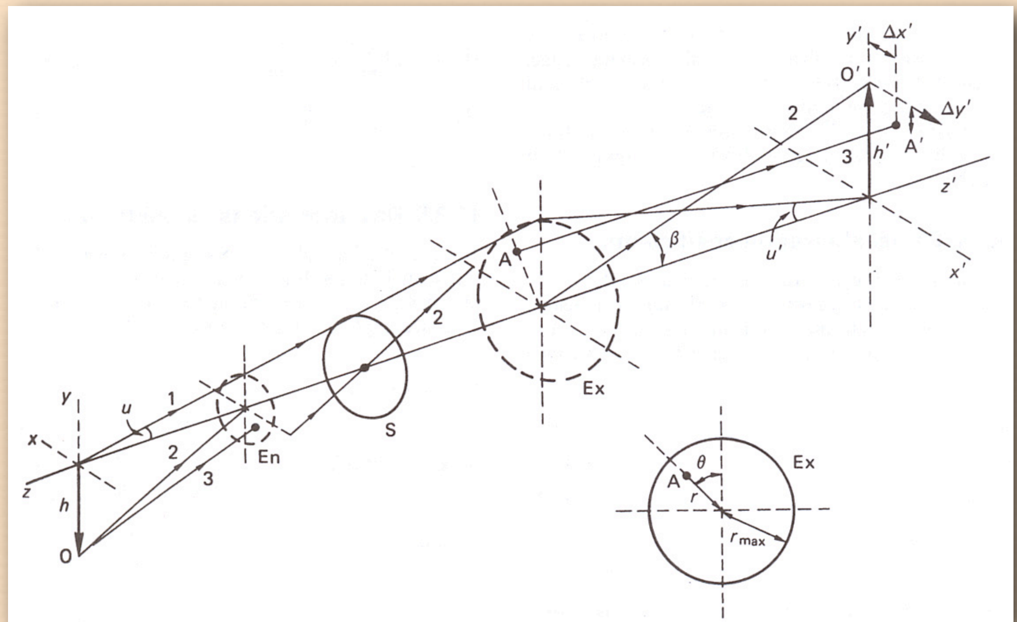

(Kingslake)

pincushion distortion

- change in magnification with image position
  (a) pincushion
  (b) barrel
- closing down the aperture does not improve this

# Algebraic formulation of monochromatic lens aberrations

Not responsible on exams for orange-tinted slides




(Smith)

- spherical aberration $a_s r^4$
- coma $a_c h' r^3 \cos\theta$
- astigmatism $a_a h'^2 r^2 \cos^2\theta$
- field curvature $a_d h'^2 r^2$
- distortion $a_t h'^3 r \cos\theta$

# Recap

- all lenses are subject to chromatic aberration
  - longitudinal appears everywhere; lateral is worse at edges
  - only longitudinal can be reduced by closing down aperture
  - both can be partly corrected using more lenses, and lateral can be partly corrected using software

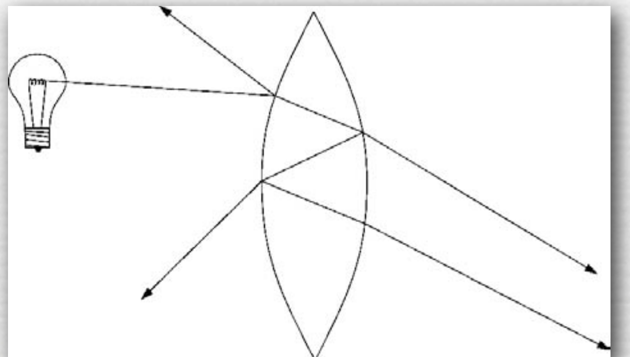
- all <u>spherical</u> lenses are subject to Seidel aberrations: spherical, coma, astigatism, field curvature, distortion
  - some appear everywhere; others only at edges
  - all but distortion can be reduced by closing down aperture
  - only distortion can be corrected completely in software

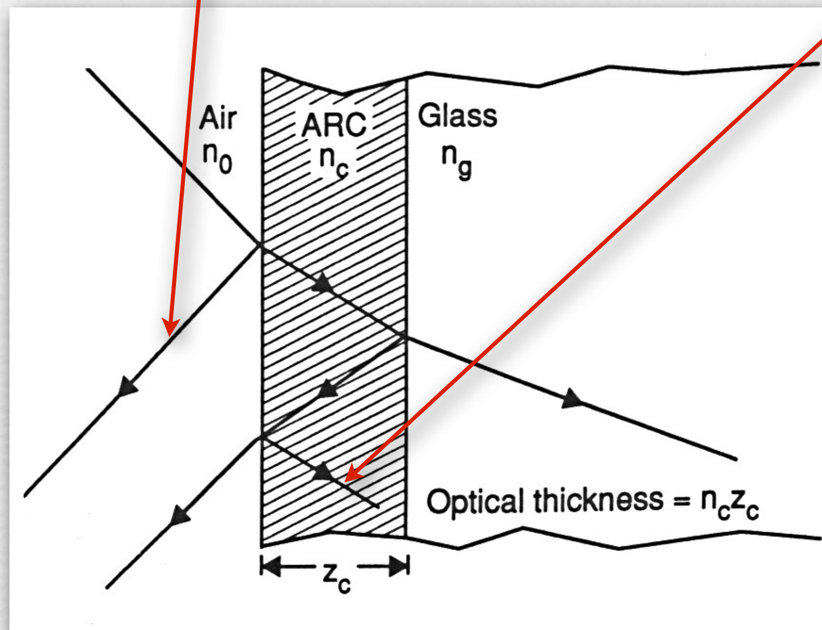
**Questions?**

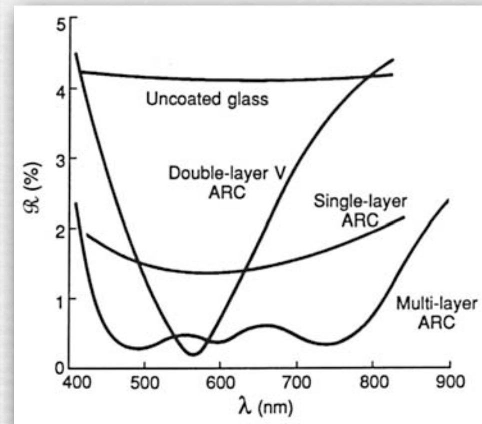
# Veiling glare

in the outermost lens we don't care about killing this reflection

but we do care about killing this one



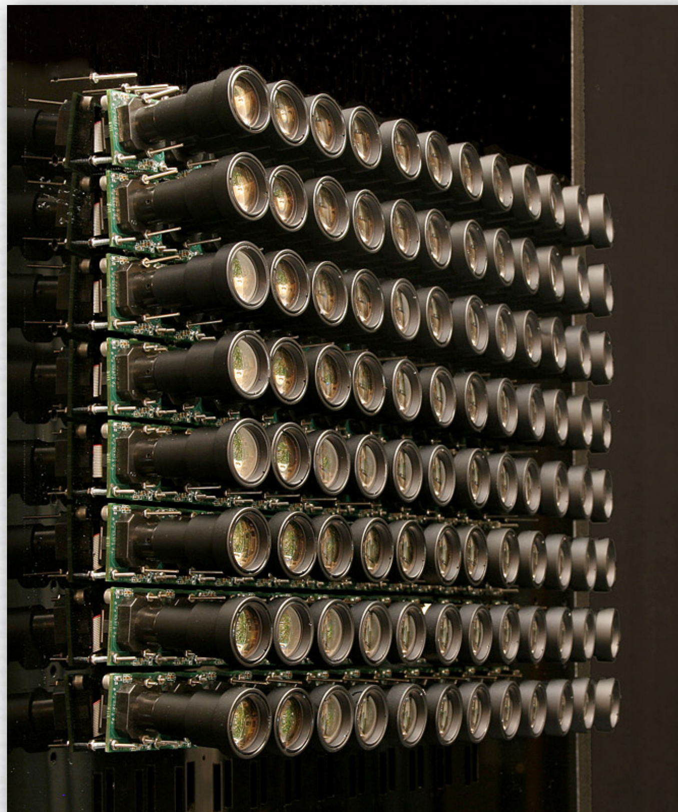



- contrast reduction caused by stray reflections
- can be reduced by anti-reflection coatings
  - based on interference, so optimized for one wavelength
  - to cover more wavelengths, use multiple coatings

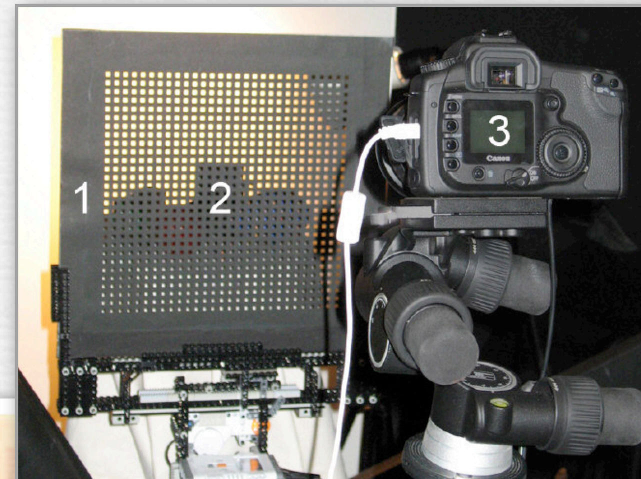
# Camera array with too much glare

Stanford
Multi-Camera
Array

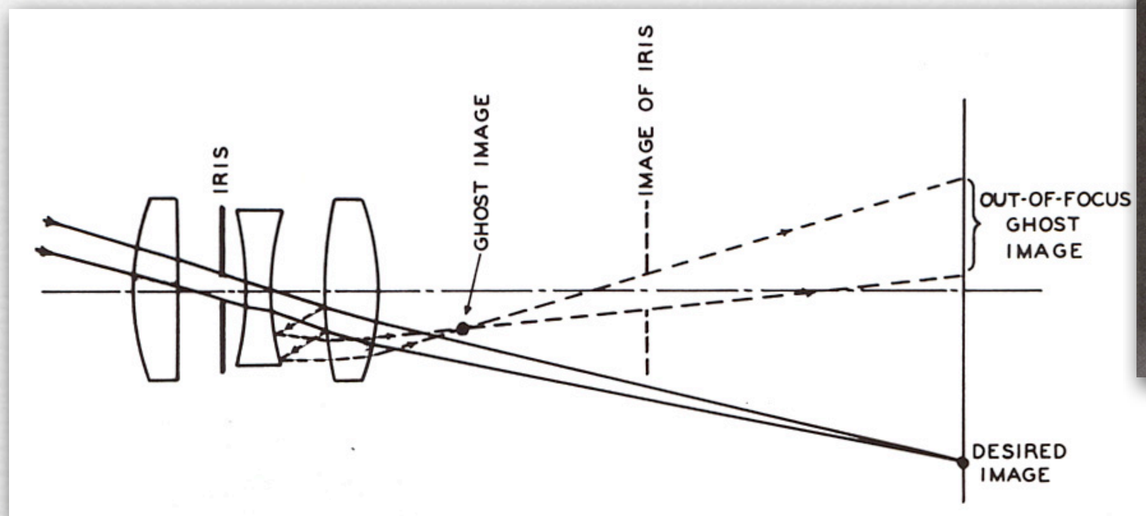


- 12 × 8 array of 600 × 800 pixel webcams = 7,200 × 6,400 pixels
- goal was highest-resolution movie camera in the world
- failed because glare in inexpensive lenses led to poor contrast

# Removing veiling glare computationally
## [Talvala, Proc. SIGGRAPH 2007]




# Flare and ghost images



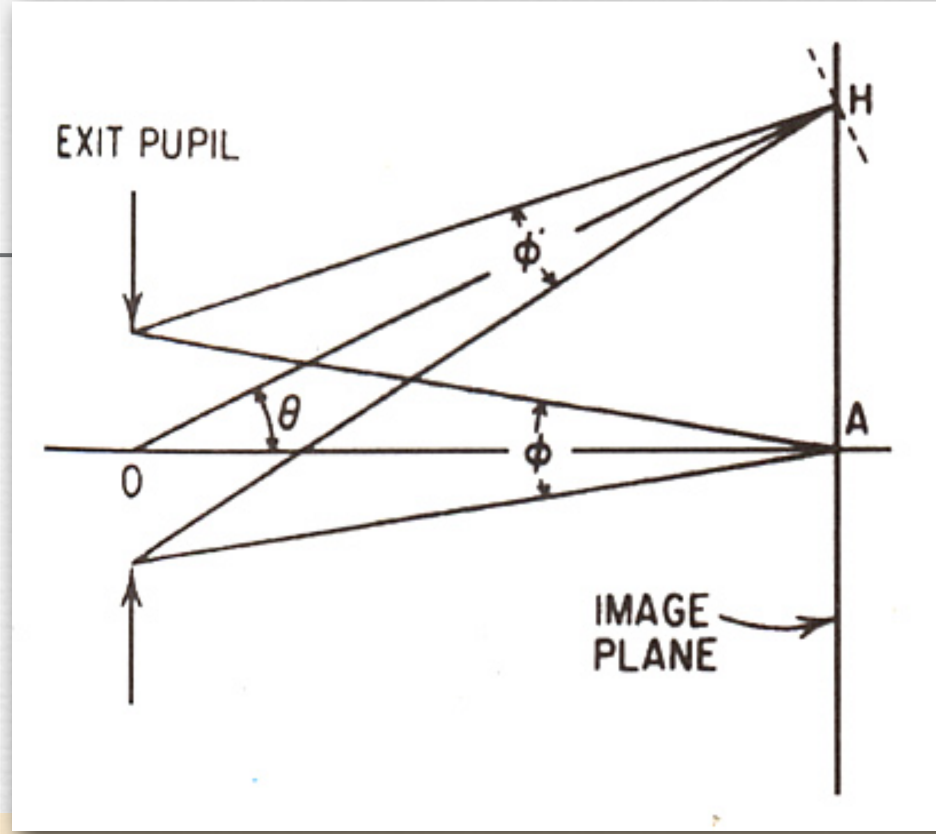

(Kingslake)

- reflections of the aperture, lens boundaries, etc., i.e. things inside the camera body
- removing these artifacts is an active area of research in computational photography
- but it's a hard problem

(Smith)

# Vignetting

(a.k.a. **natural** vignetting)




- irradiance is proportional to projected area of aperture as seen from pixel on sensor, which drops as cos θ
- irradiance is proportional to projected area of pixel as seen from aperture, which also drops as cos θ
- irradiance is proportional to distance$^2$ from aperture to pixel, which rises as 1/cos θ
- combining all these effects, light drops as $\cos^4 \theta$

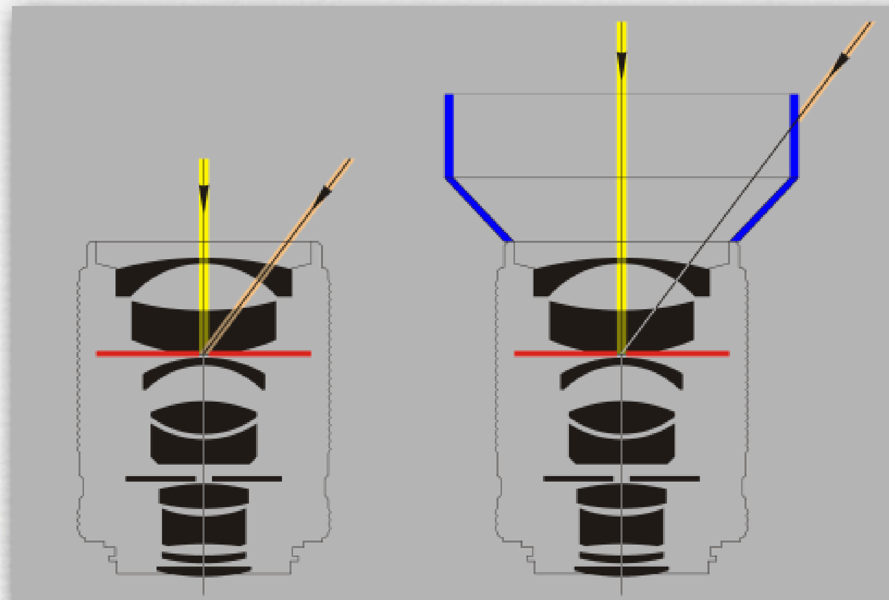
# Other sources of vignetting

f/1.4 f/5.6

axial

semifield


(toothwalker.org)



**optical** vignetting
from multiple lens elements,
especially at wide apertures

**mechanical** vignetting
from add-on lens hoods
(or filters or fingers)

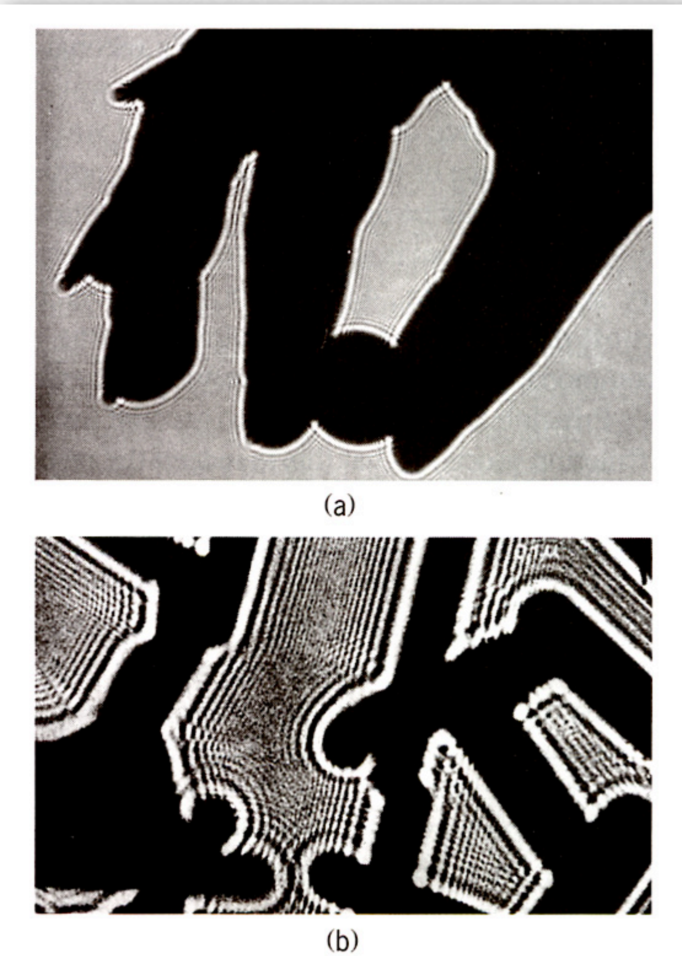
- **pixel** vignetting due to shadowing inside each pixel
(we'll come back to this)

# Examples


(toothwalker.org) (toothwalker.org) (wikipedia)

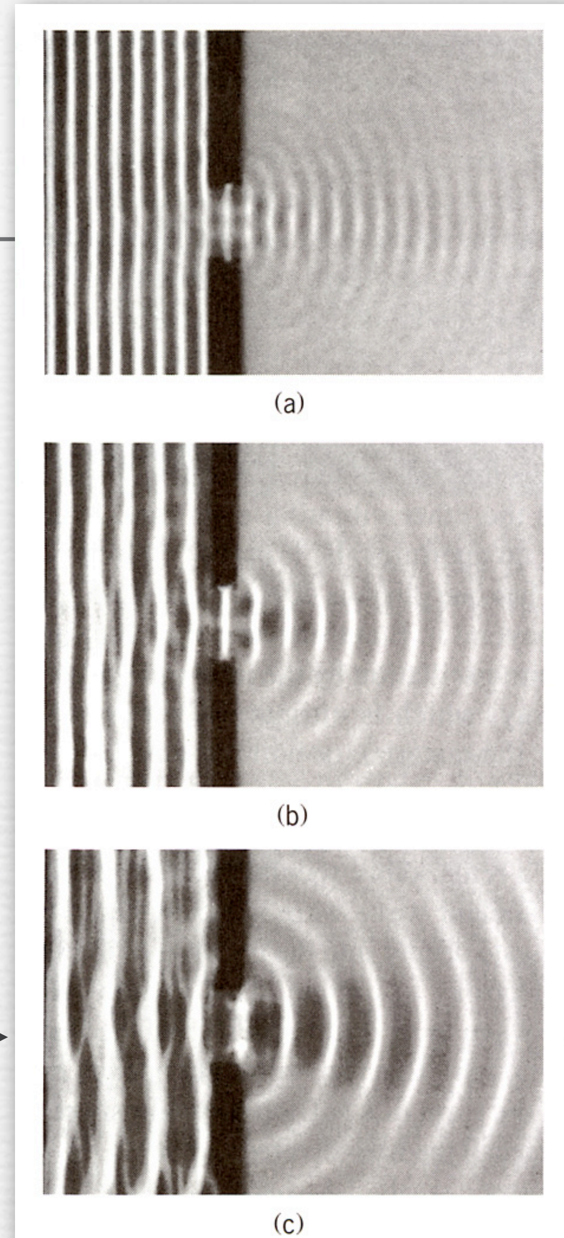
- vignetting causes falloff in brightness towards edges of image
- vignetting affects the *bokeh* of out-of-focus features
- vignetting is correctable in software (except for bokeh effects), but boosting pixel values worsens noise
- vignetting can be applied afterwards, for artistic effects

# Diffraction




(Hecht)

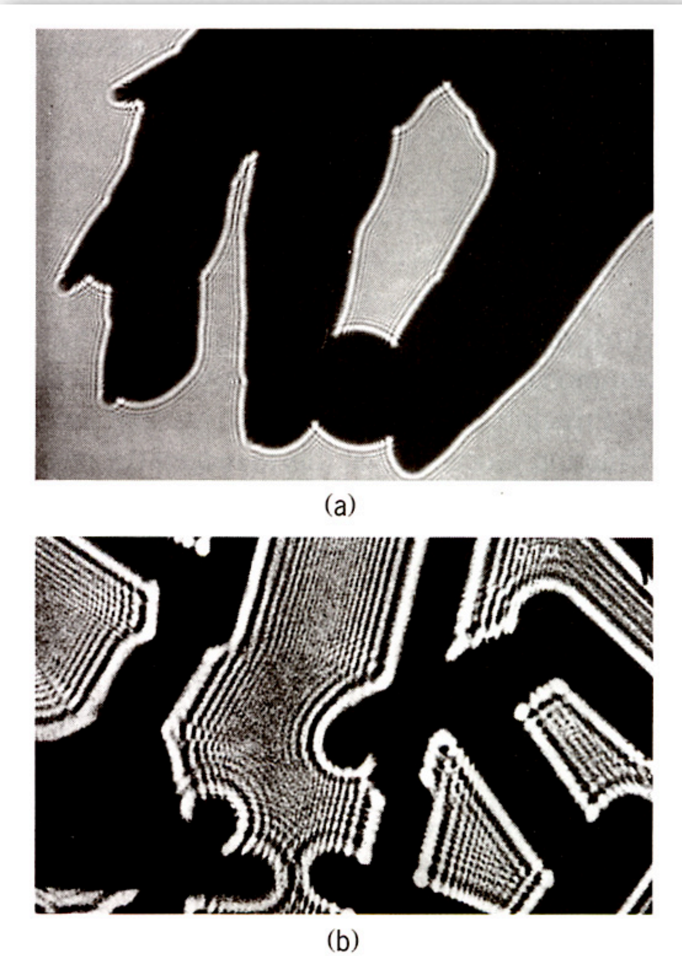
illuminated by a (spread-out) laser beam & recorded directly on film




varying the wavelength of waves passing through a slit in a ripple tank

- as wavelength decreases in the ripple tank, propagation becomes more ray-like

# Diffraction




(Hecht)

illuminated by a (spread-out) laser beam & recorded directly on film

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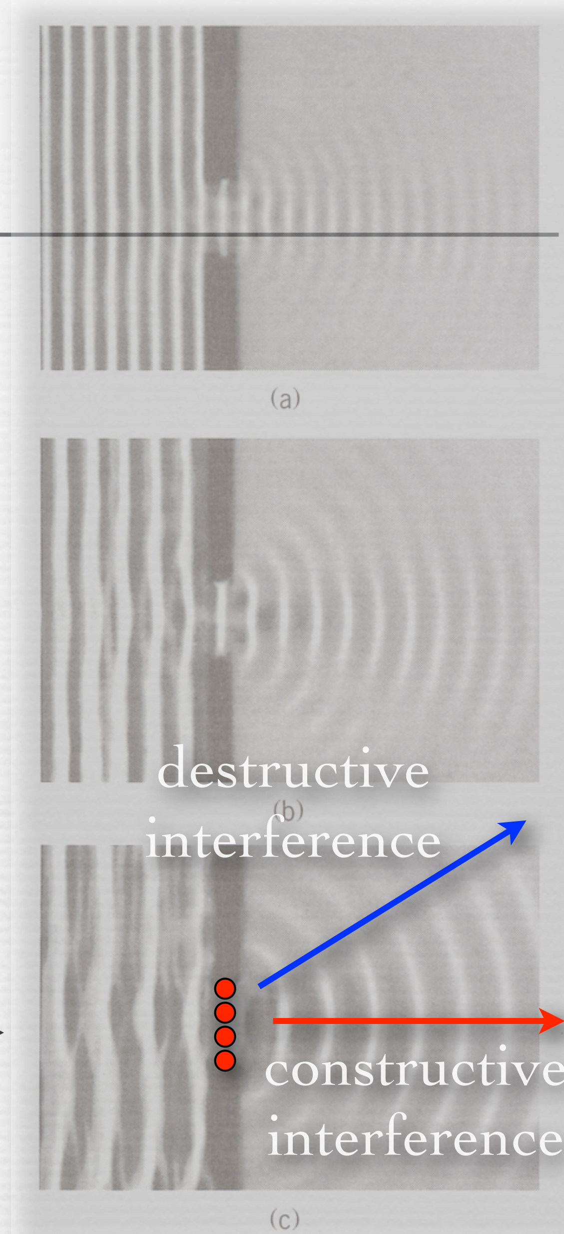


- ✦ as wavelength decreases in the ripple tank, propagation becomes more ray-like

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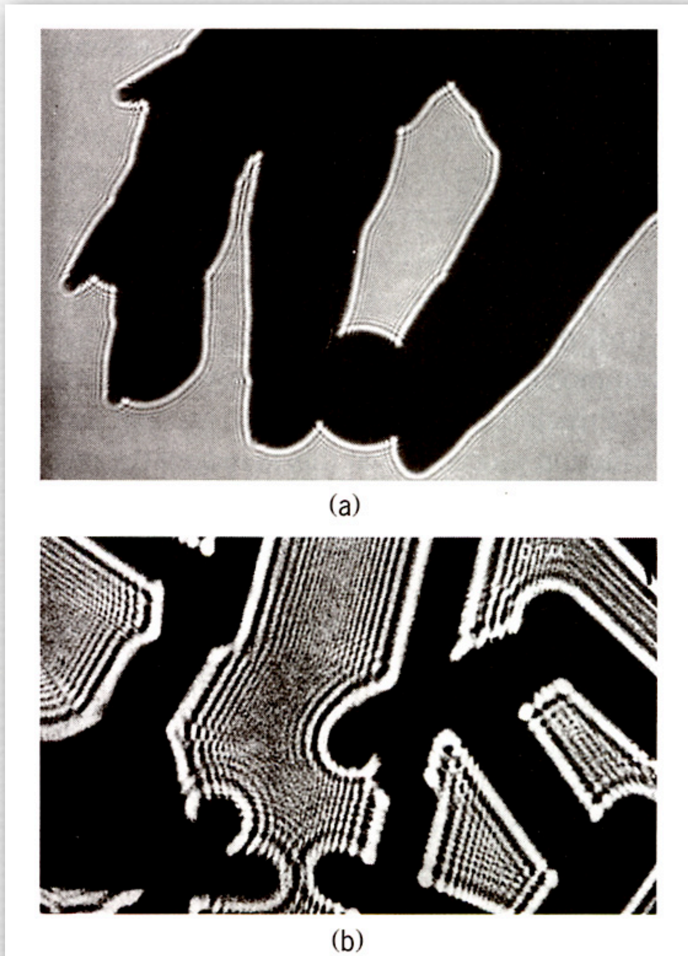


(Hecht)

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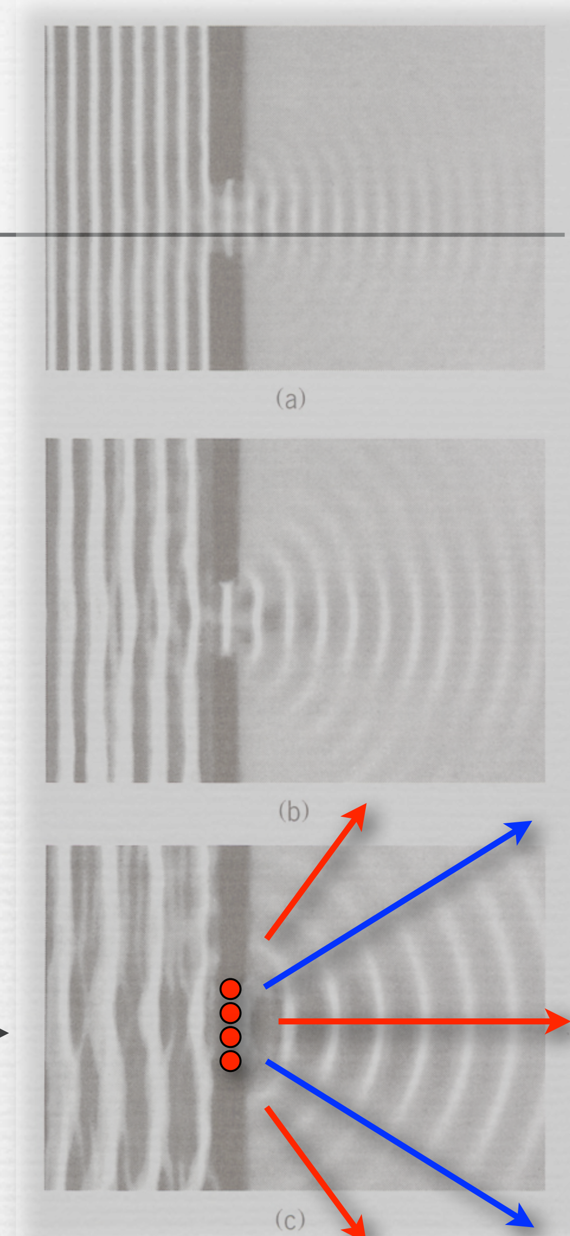


varying the wavelength of waves passing through a slit in a ripple tank

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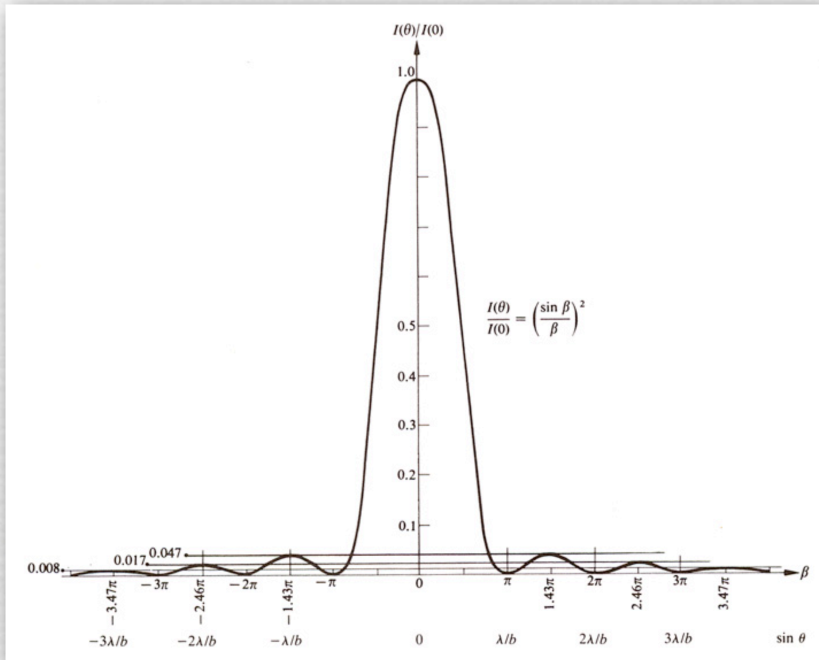
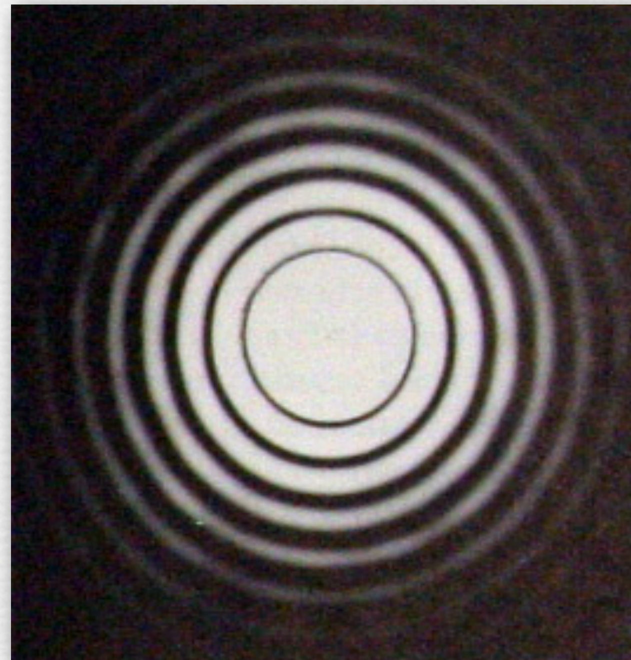
# Airy rings




diffraction from a slit



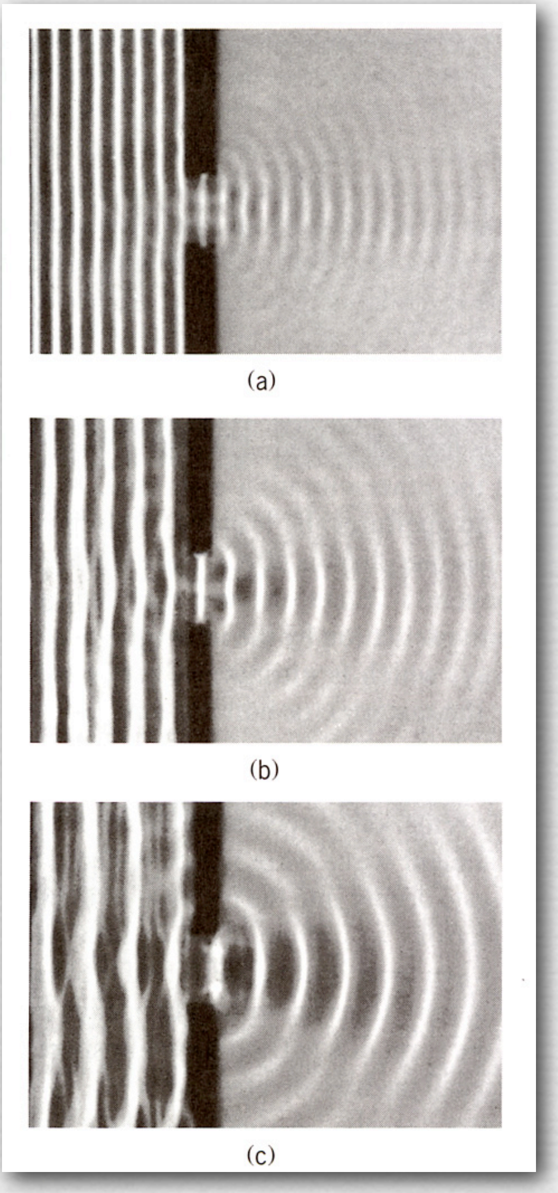
(Hecht)

diffraction from a circular aperture: Airy rings

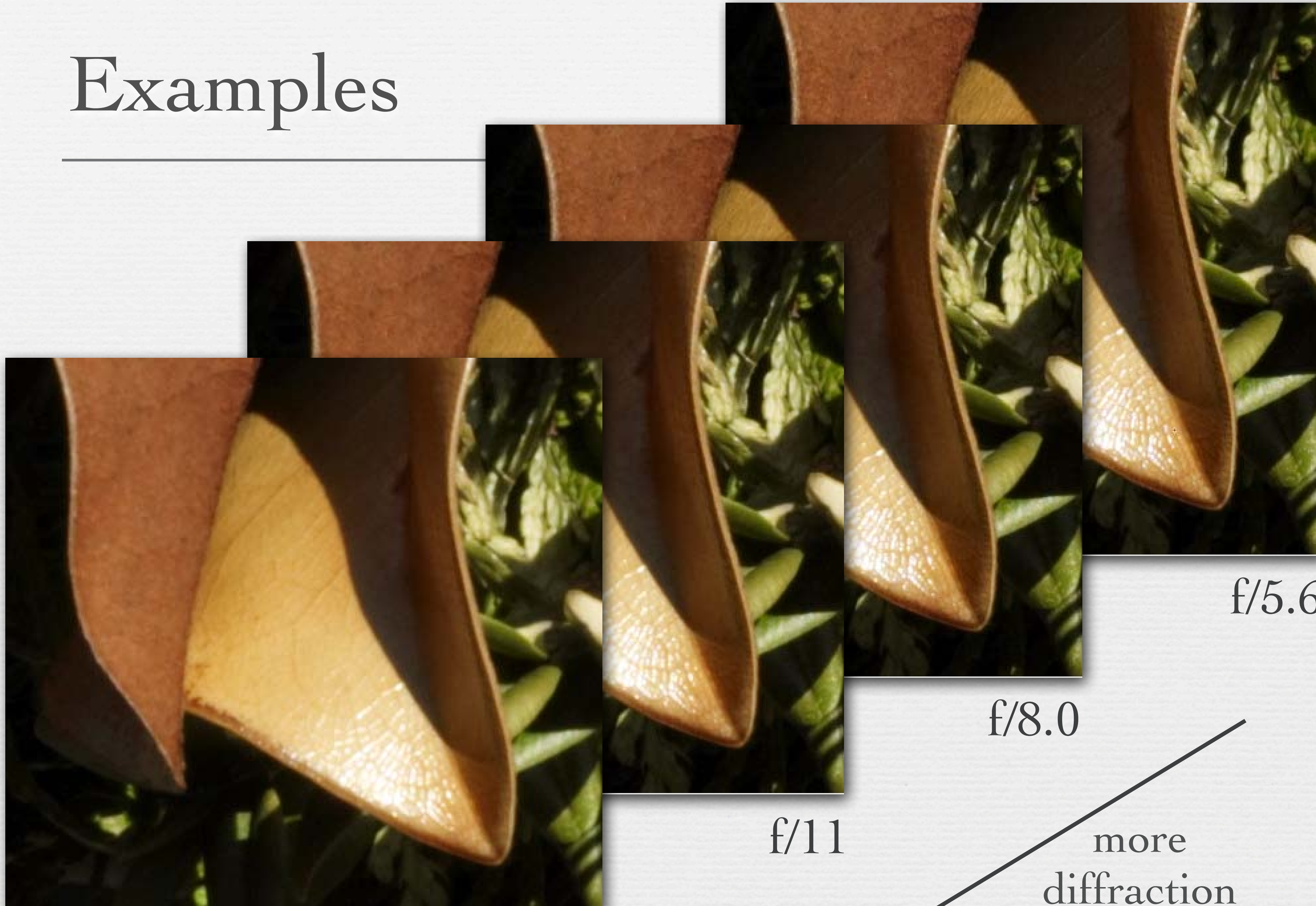
- if the illumination were a laser, a lens would produce this pattern
- but considering all wavelengths, the dark rings vanish, leaving a blur

# Diffraction in photographic cameras

- well-corrected lenses are called *diffraction-limited*
- the smaller the aperture ($A$) (or the longer the wavelength), the larger the diffraction blur
- the longer the distance to the sensor ($f$), the larger the blur
- thus, the size of the blur varies with $N = f / A$



# Examples



f/5.6

f/8.0

f/11

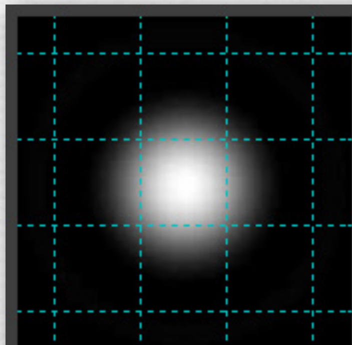
f/22

more diffraction

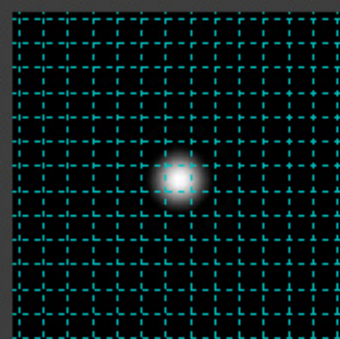
(luminous-landscape.com)

# Diffraction in photographic cameras

- the smaller the pixels, the more of them the pattern covers
  - if the pattern spans >> 1 pixel, we begin to complain



| Aperture | Camera Type | Pixel Area |
|---|---|---|
| f/2.0 | Canon EOS 1D | 136. $\mu m^2$ |
| f/2.8 | Canon EOS 1Ds | 77.6 $\mu m^2$ |
| f/4.0 | Canon EOS 1DMkII / 5D | 67.1 $\mu m^2$ |
| f/5.6 | Nikon D70 | 61.1 $\mu m^2$ |
| f/8.0 | Canon EOS 10D | |
| f/11 | Canon EOS 1DsMkII | |
| f/16 | Canon EOS 20D / 35 | |
| f/22 | Nikon D2X | |
| f/32 | Canon PowerShot G | |



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| f/5.6 | Nikon D70 | 61.1 $\mu m^2$ |
| f/8.0 | Canon EOS 10D | 54.6 $\mu m^2$ |
| f/11 | Canon EOS 1DsMkII | 52.0 $\mu m^2$ |
| f/16 | Canon EOS 20D / 350D | 41.2 $\mu m^2$ |
| f/22 | Nikon D2X | 30.9 $\mu m^2$ |
| f/32 | Canon PowerShot G6 | 5.46 $\mu m^2$ |

(http://www.cambridgeincolour.com/tutorials/diffraction-photography.htm)

# The Abbe diffraction limit

$$d = \frac{.61\,\lambda}{NA} \approx 1.2\,N\,\lambda$$

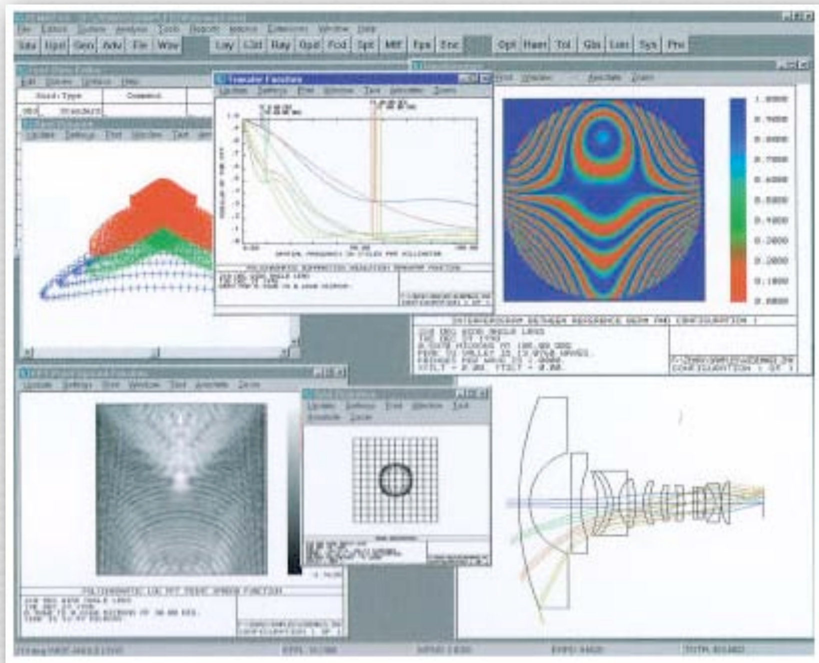
- where
  - $\lambda$ = wavelength
  - $NA$ = numerical aperture $\approx 1 / 2N$
- Example: iPhone 4 when looking at green
  - $\lambda$ = 550nm
  - $N$ = f/3
  - $d$ = 2μ
  - pixels are 1.75μ wide, so the iPhone 4 would be roughly diffraction-limited if its lenses were free of aberrations

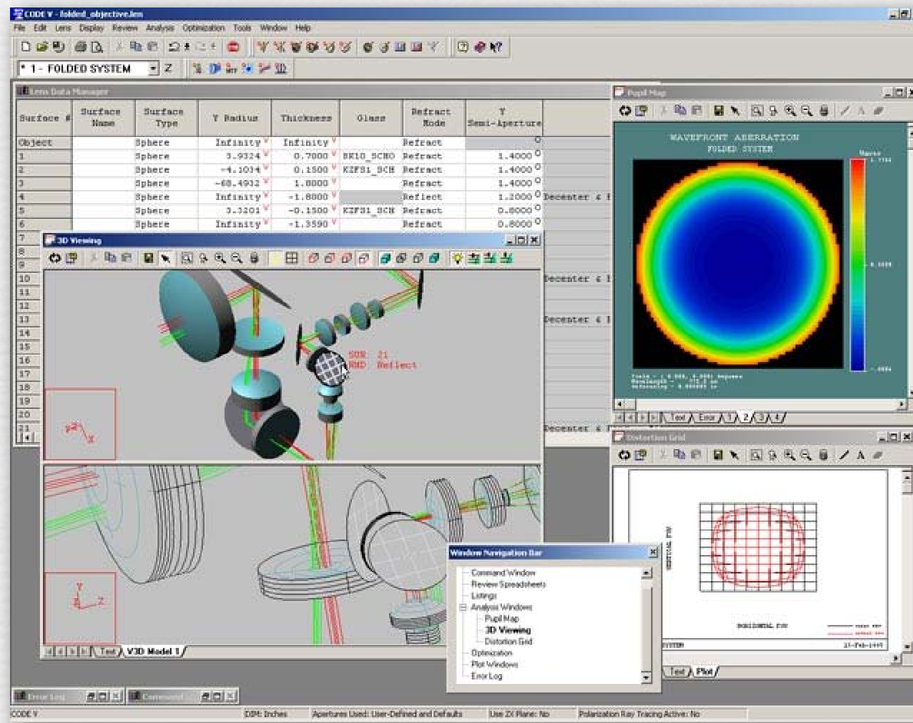
# Recap

- all optical systems suffer from veiling glare
  - anti-reflection coatings help
- all optical systems suffer from flare and ghosts
  - don't point your camera at bright lights; use lens hoods
- vignetting arises from many sources
  - natural - falloff at the edges of wide sensors
  - optical - caused by apertures, lens barrels
  - mechanical - caused by wrong lens hoods, hands, straps
  - pixel - caused by shadowing inside pixel structures
- diffraction - blur that varies with $N = f / A$
  - avoid F-numbers above f/13 (for full-frame camera)
  - subjective image quality depends on both sharpness and contrast

**Questions?**

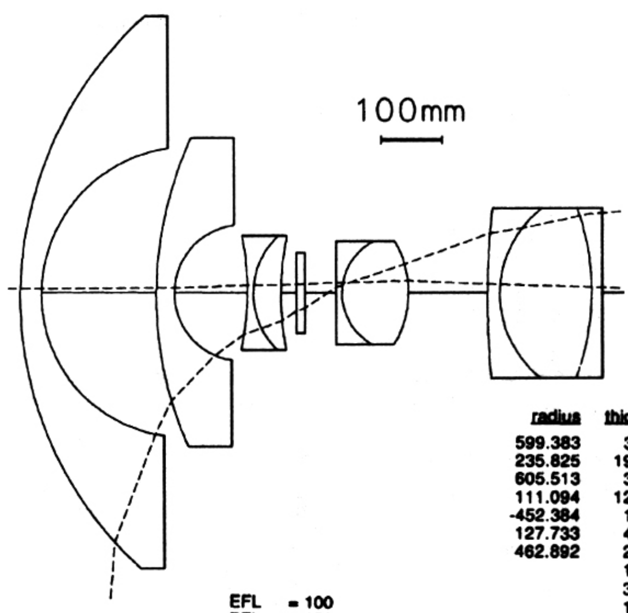
# Lens design software






- ✦ uses optimization to make good recipes better

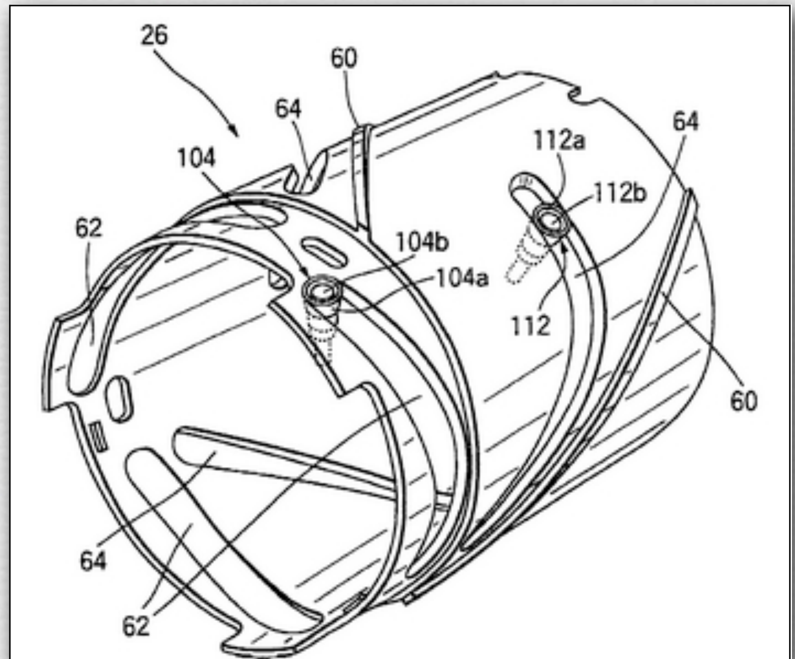
# Lens catalogs and patents




FISHEYE F/8 90DEGHFOV
MIYAMOTO JOSA 1964

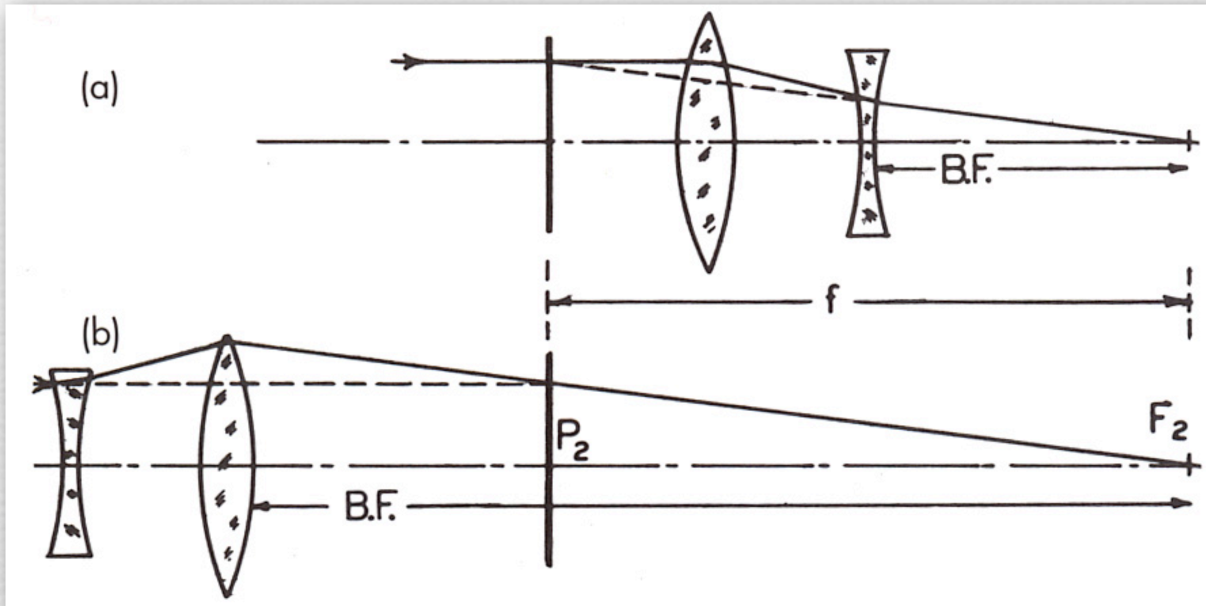
| radius | thickness | mat'l | index | V-no | sa |
|---|---|---|---|---|---|
| 599.383 | 35.030 | BK7 | 1.517 | 64.2 | 448.4 |
| 235.825 | 190.161 | air | | | 234.0 |
| 605.513 | 30.025 | FK5 | 1.487 | 70.4 | 251.8 |
| 111.094 | 120.102 | air | | | 110.1 |
| -452.384 | 10.008 | FK5 | 1.487 | 70.4 | 93.5 |
| 127.733 | 45.038 | SF56 | 1.785 | 26.1 | 93.5 |
| 462.892 | 25.021 | air | | | 93.5 |
| | 15.013 | K3 | 1.518 | 59.0 | 65.4 |
| | 36.281 | air | | | 65.5 |
| | 13.762 | air | | | 15.8 |
| 38507.649 | 10.008 | SF56 | 1.785 | 26.1 | 84.1 |
| 95.081 | 110.093 | LAF2 | 1.744 | 44.7 | 84.1 |
| -162.638 | 130.110 | air | | | 84.1 |
| 1376.167 | 20.017 | SF56 | 1.785 | 26.1 | 139.0 |
| 177.275 | 150.127 | BSF52 | 1.702 | 41.0 | 139.0 |
| -400.339 | 18.766 | BASF6 | 1.668 | 41.9 | 139.0 |
| -337.536 | 150.119 | air | | | 139.0 |

EFL = 100
BFL = 150.1
NA = -0.0626 (F/8.0)
GIH = 133.60 (HFOV=85.40)
PTZ/F = 49.15
VL = 959.56
OD infinite conjugate




- hard to find optical recipe for commercial camera lenses

# Lens combinations: telephoto

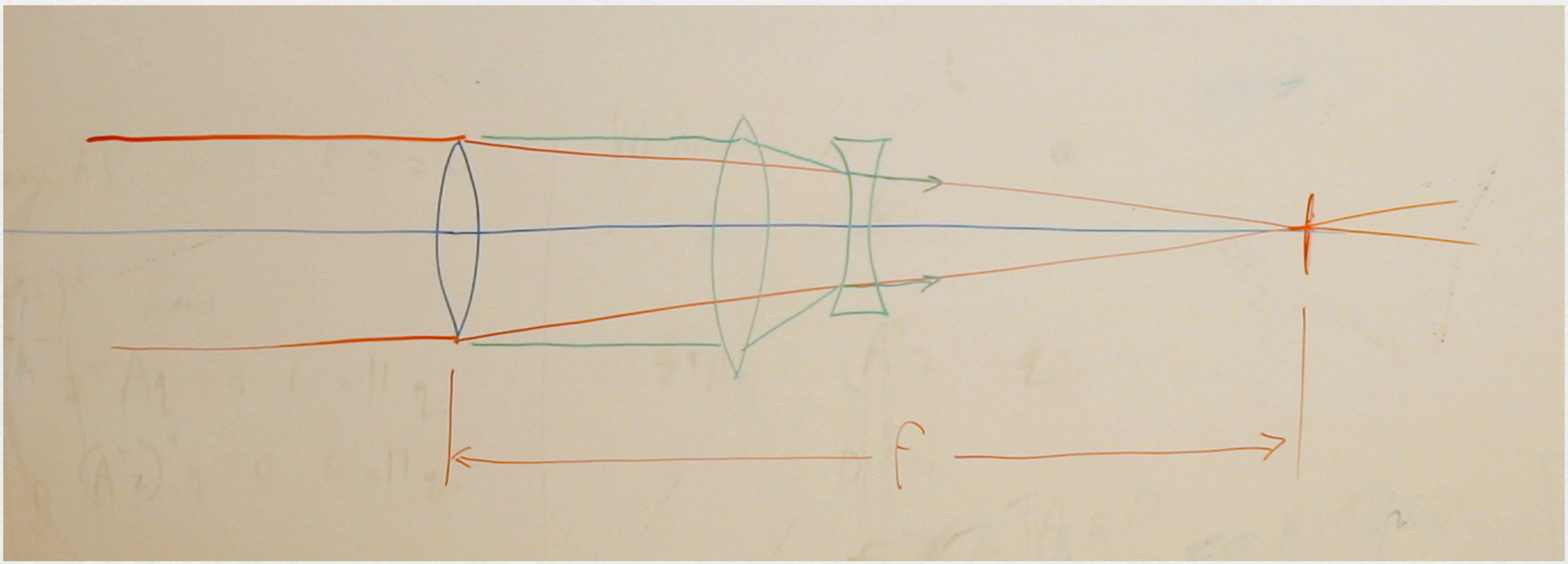



(Kingslake)

- telephoto (a) reduces the back focal distance B.F. relative to *f*
  - for long focal length lenses, to reduce their physical size
- reversed telephoto (b) increases B.F. relative to *f*
  - for wide-angle lenses, to ensure room for the reflex mirror

# Telephoto lens

- the blue lens is replaced with the two green ones, thereby reducing the physical size of the lens assembly, while preserving its focal length (hence magnification)

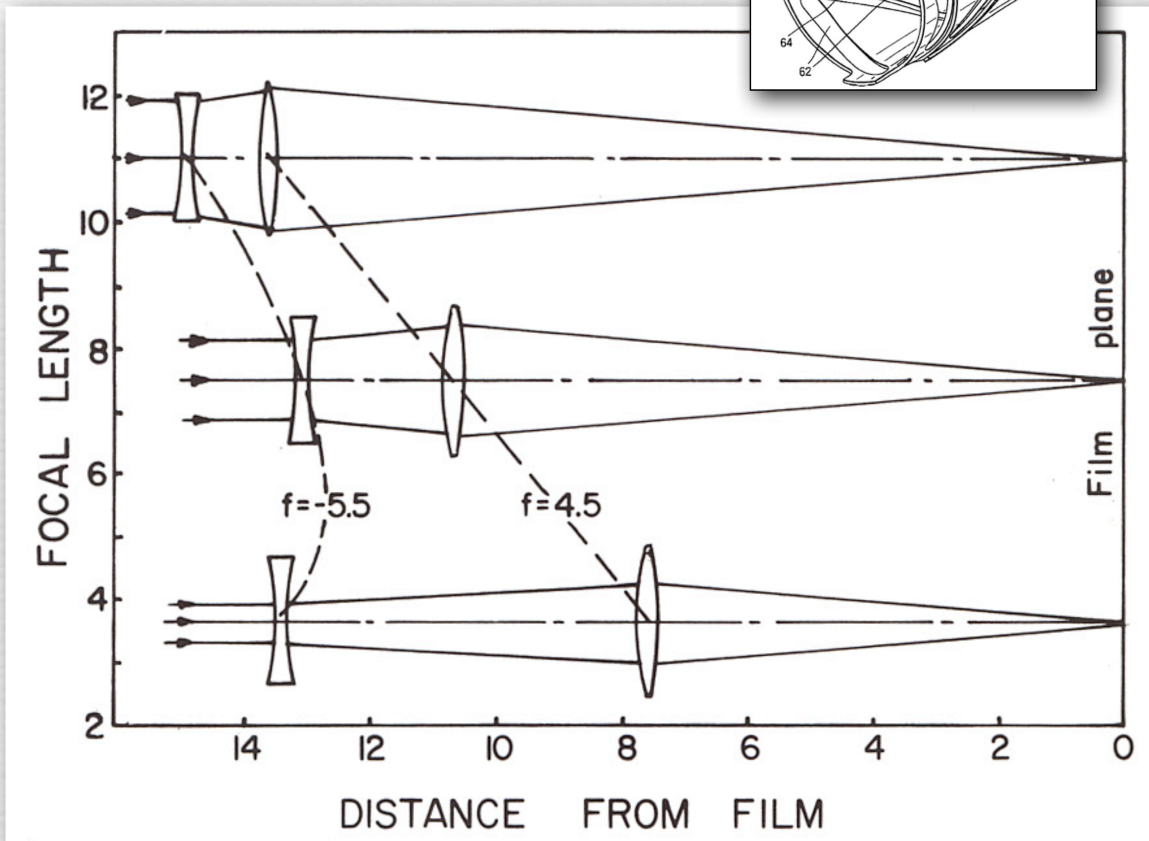


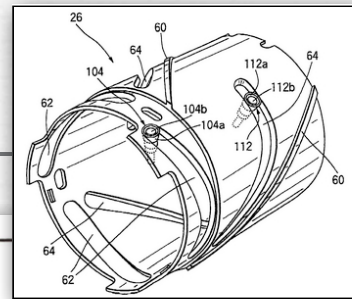
# Lens combinations: telephoto

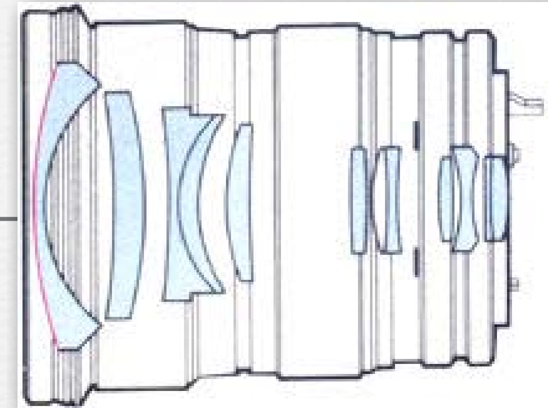
Nikon 500mm telephoto

Opteka 500mm <u>non</u>-telephoto

# Lens combinations: zoom





Canon FD 24-35mm f/3.5 L manual focus lens

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/zoom.html

- called *optically compensated zoom*, because the in-focus plane stays (more or less) stationary as you zoom
- to change focus, you move both lenses together

# Recap

- telephoto lenses separate focal length & back focal distance
  - for long focal length lenses, to reduce their physical size
  - for wide-angle lenses, to ensure room for the reflex mirror
- most modern zoom lenses are focus-compensated
  - as you zoom, they stay in focus

**Questions?**

# Slide credits

- Steve Marschner
- Fredo Durand


- Cole, A., *Perspective*, Dorling Kindersley, 1992.
- Kemp, M.,*The Science of Art*,Yale University Press, 1990.
- Hecht, E., *Optics* (4th ed.), Pearson / Addison-Wesley, 2002.
- Renner, E., *Pinhole Photography* (2nd ed.), Focal Press, 2000.
- London, Stone, and Upton, *Photography* (9th ed.), Prentice Hall, 2008.
- D'Amelio, J., *Perspective Drawing Handbook*, Tudor Press, 1964.
- Dubery, F., Willats, J., *Perspective and other drawing systems*, Van Nostrand Reinhold, 1972.
- Kingslake, R. *Optics in Photography*, SPIE Press, 1992.
- Pamplona et al., "NETRA: Interactive Display for Estimating Refractive Errors and Focal Range", *Proc. SIGGRAPH 2010.*
- Son, H.S. et al., "Design of a spherical focal surface using close-packed relay optics", *Optics Express* 19(17), 2011.
- http://dpreview.com