Optics I:
lenses and apertures

CS 178, Spring 2010

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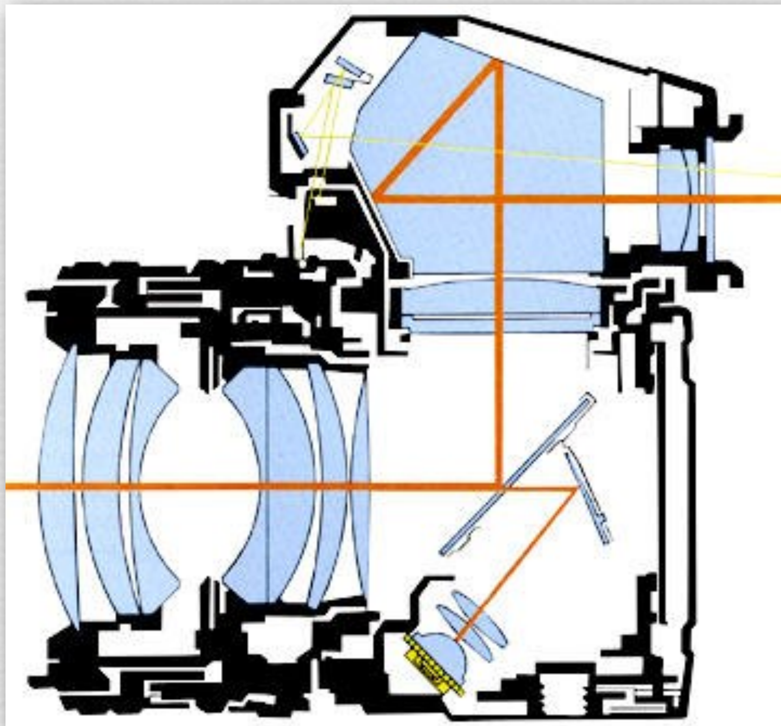
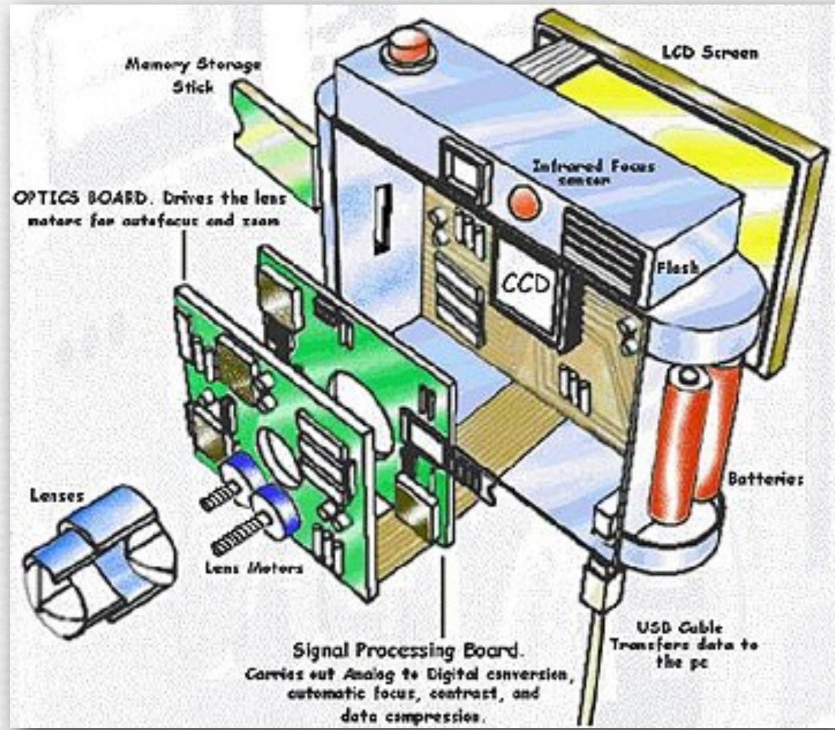
Outline

- why study lenses?
- thin lenses
  - graphical constructions, algebraic formulae
- thick lenses
  - lenses and perspective transformations
- depth of field
- aberrations & distortion
- vignetting, glare, and other lens artifacts
- diffraction and lens quality
- special lenses
  - telephoto, zoom

Cameras and their lenses

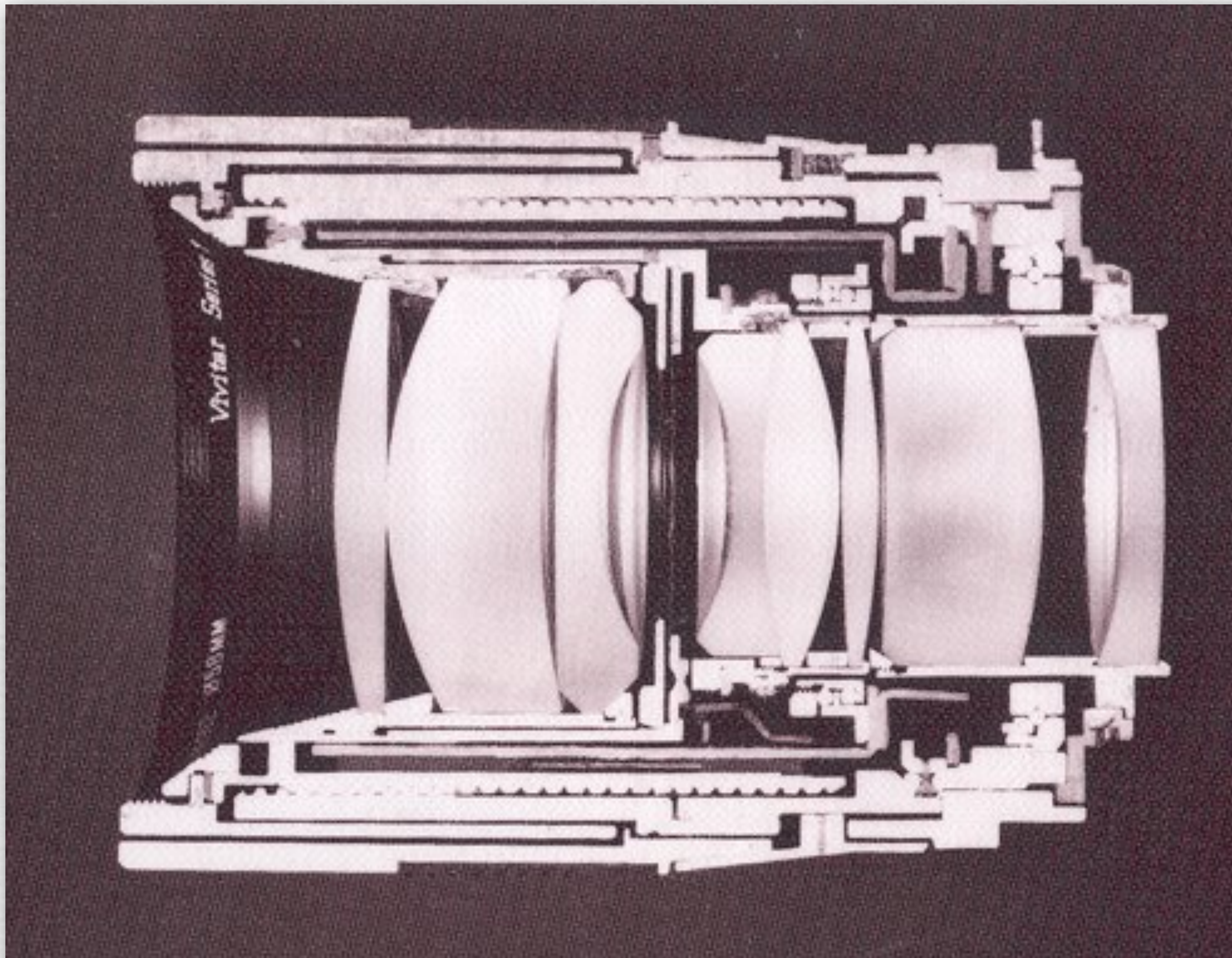


single lens reflex
(SLR) camera



digital still camera (DSC),
i.e. point-and-shoot

Cutaway view of a real lens



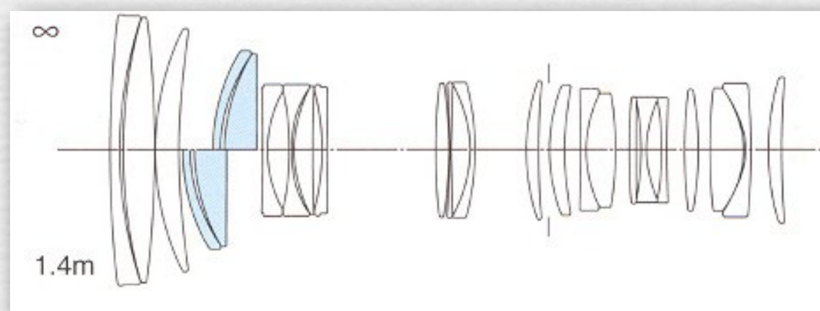
Vivitar Series 1 90mm f/2.5
Cover photo, Kingslake, *Optics in Photography*

Lens quality varies

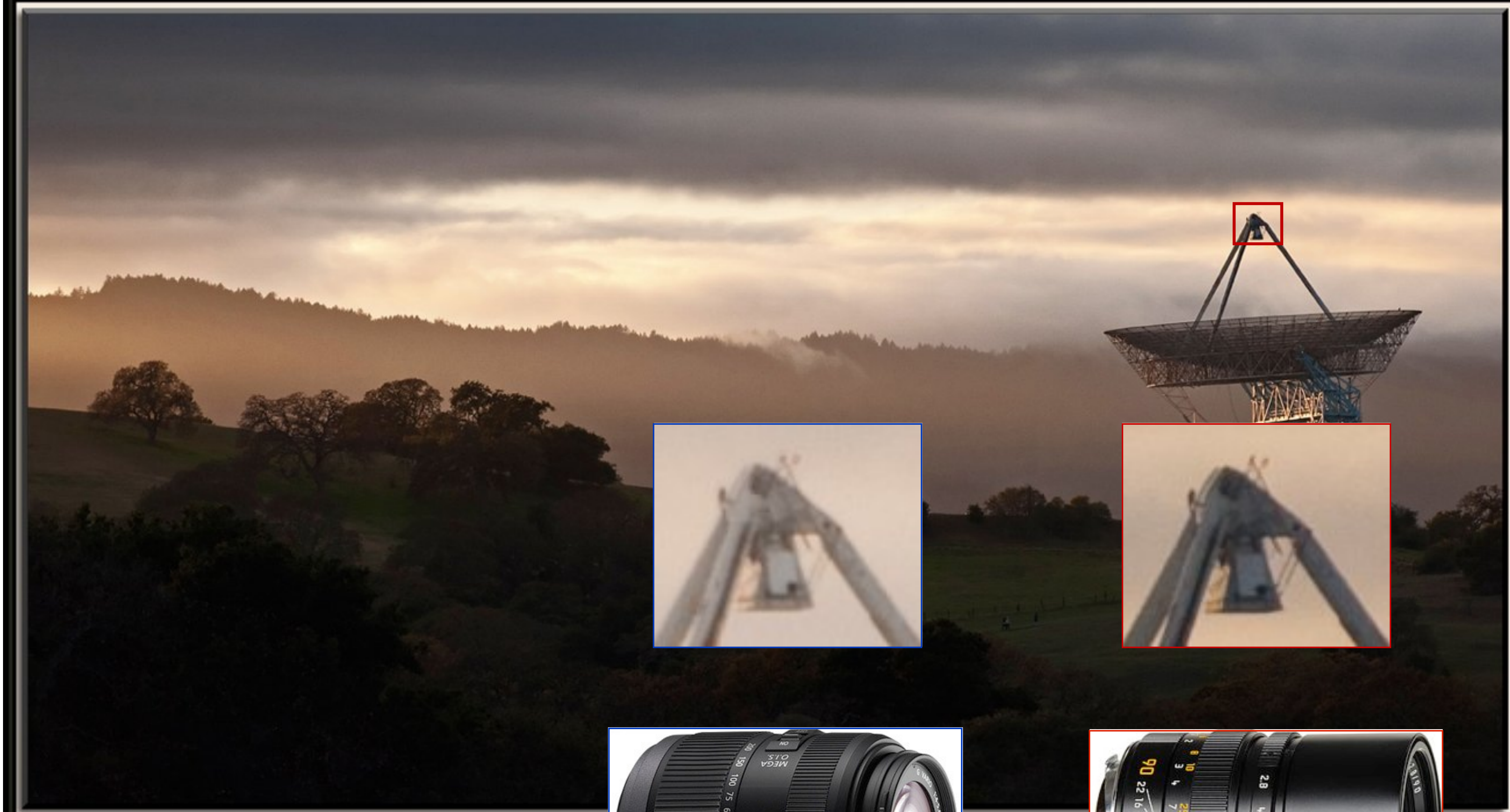
- Why is this toy so expensive?
  - EF 70-200mm f/2.8L IS USM
  - $1700

- Why is it better than this toy?
  - EF 70-300mm f/4-5.6 IS USM
  - $550

- Why is it so complicated?



(Canon)



Stanford Big Dish

Panasonic GF1

Panasonic 45-200/4-5.6 zoom, at 200mm f/4.6
$300

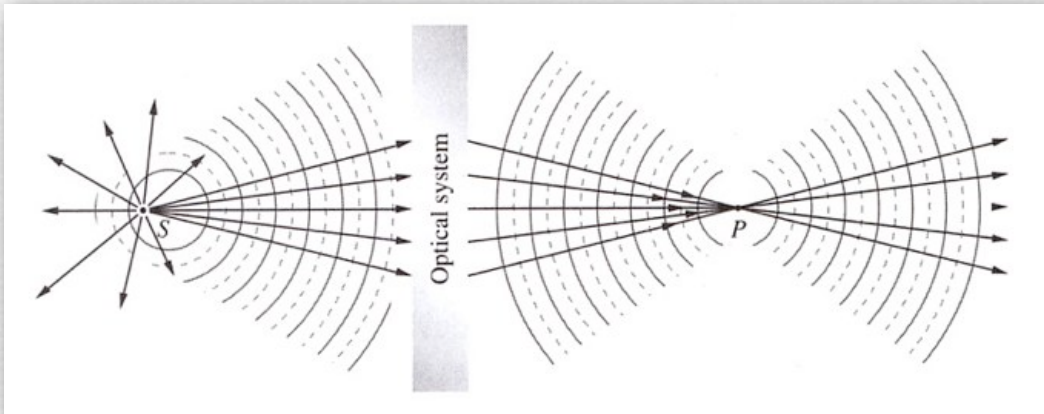
Leica 90mm/2.8 Elmarit-M prime, at f/4
$2000

Zoom lens versus prime lens

Canon 100-400mm/4.5-5.6
zoom, at 300mm and f/5.6
$1600

Canon 300mm/2.8
prime, at f/5.6
$4300

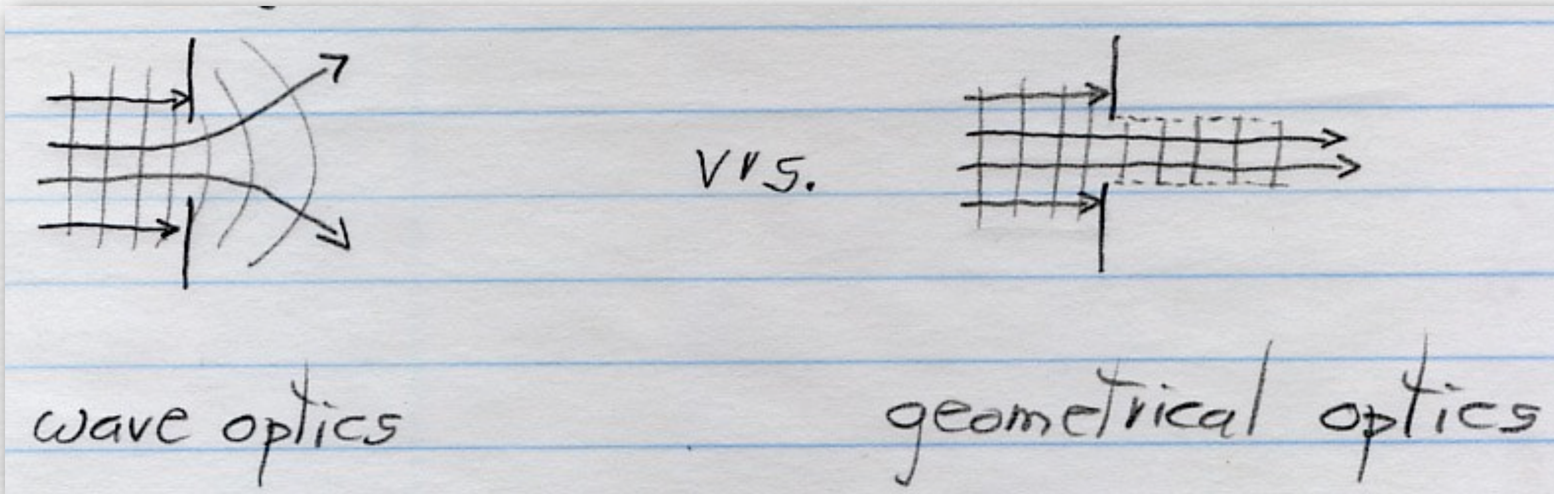
Physical versus geometrical optics



(Hecht)

- light can be modeled as traveling waves
- the perpendiculars to these waves can be drawn as rays
- diffraction causes these rays to bend, e.g. at a slit
- *geometrical optics* assumes
  - $\lambda \rightarrow 0$
  - no diffraction
  - in free space, rays are straight (a.k.a. rectilinear propagation)

Physical versus geometrical optics (contents of whiteboard)



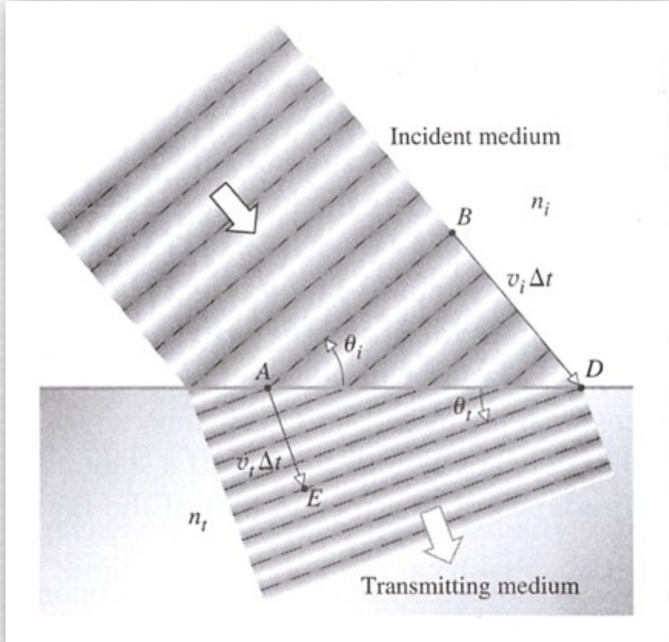
- in geometrical optics, we assume that rays do not bend as they pass through a narrow slit
- this assumption is valid if the slit is much larger than the wavelength
- physical optics is a.k.a. wave optics

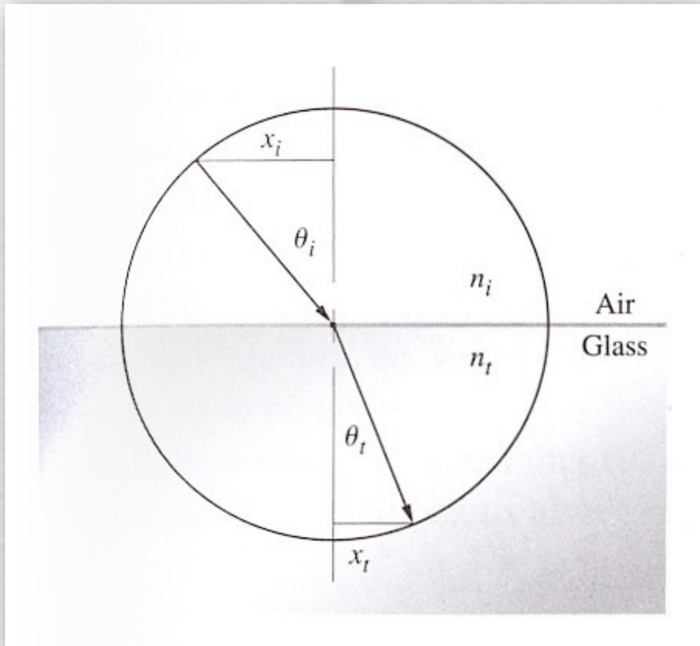
Sudden jump to n sin i notation clarified on 5/1/10.

later we will use
n sin i = n' sin i'
(for indices n and n'
and i and i' in radians)

Snell's law of refraction

(Hecht)





$$\frac{x_i}{x_t} = \frac{\sin\theta_i}{\sin\theta_t} = \frac{n_t}{n_i}$$

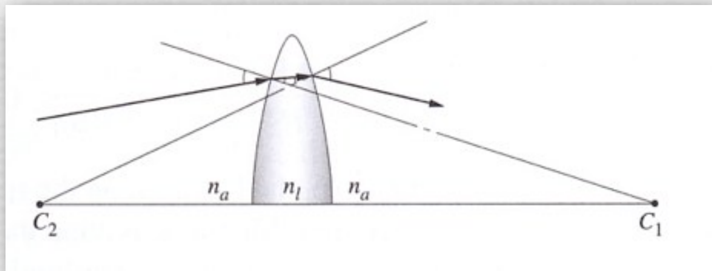
- as waves change speed at an interface, they also change direction
- index of refraction *n* is defined as the ratio between the speed of light in a vaccum / speed in some medium

Typical refractive indices (n)

- air = 1.0
- water = 1.33
- glass = 1.5 - 1.8

mirage due to changes in the index of refraction of air with temperature

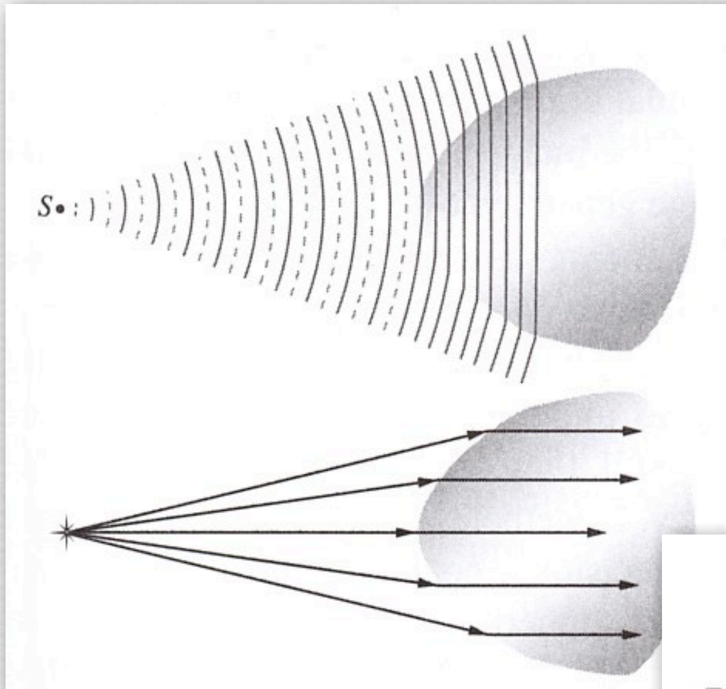
- when transiting from air to glass, light bends towards the normal
- when transiting from glass to air, light bends away from the normal



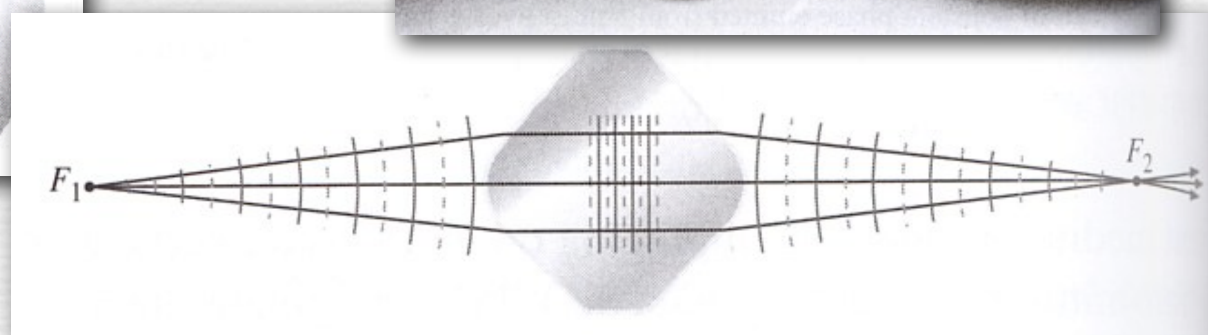
(Hecht)

- light striking a surface perpendicularly does not bend

Q. What shape should an interface be to make parallel rays converge to a point?



(Hecht)

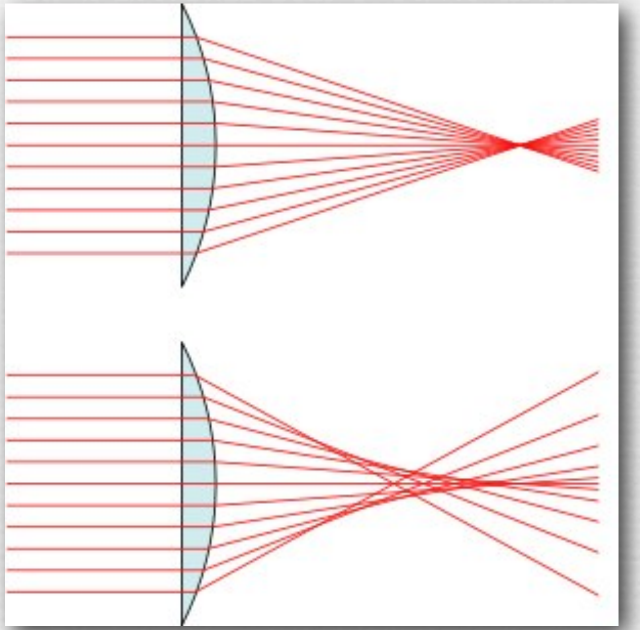


A. a hyperbola

- so lenses should be hyperbolic!

Spherical lenses

(Hecht)

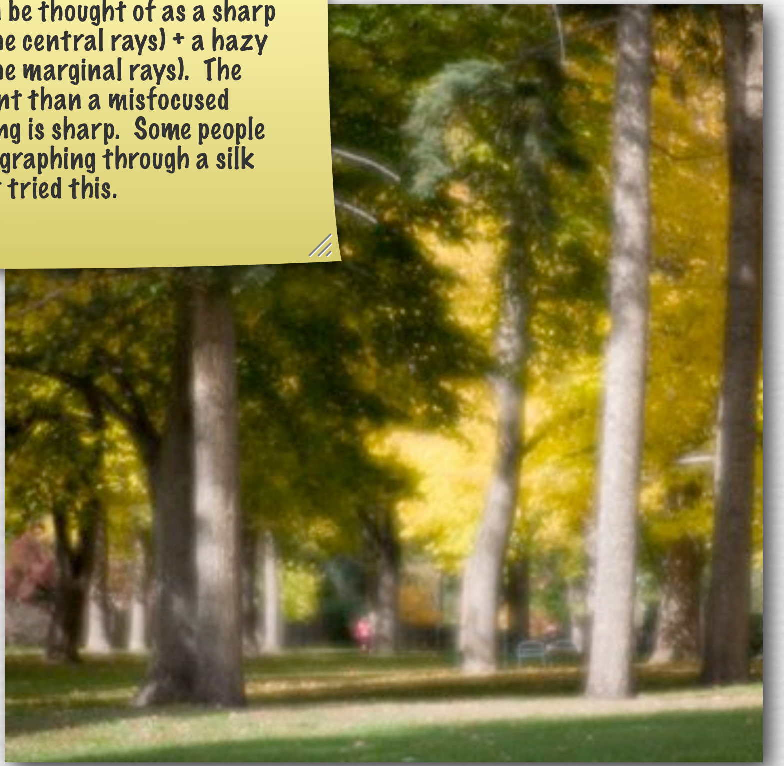


(wikipedia)

- two roughly fitting curved surfaces ground together will eventually become spherical
- spheres don't bring parallel rays to a point
  - this is called *spherical aberration*
  - nearly axial rays *(paraxial rays)* behave best

Examples of spherical aberration

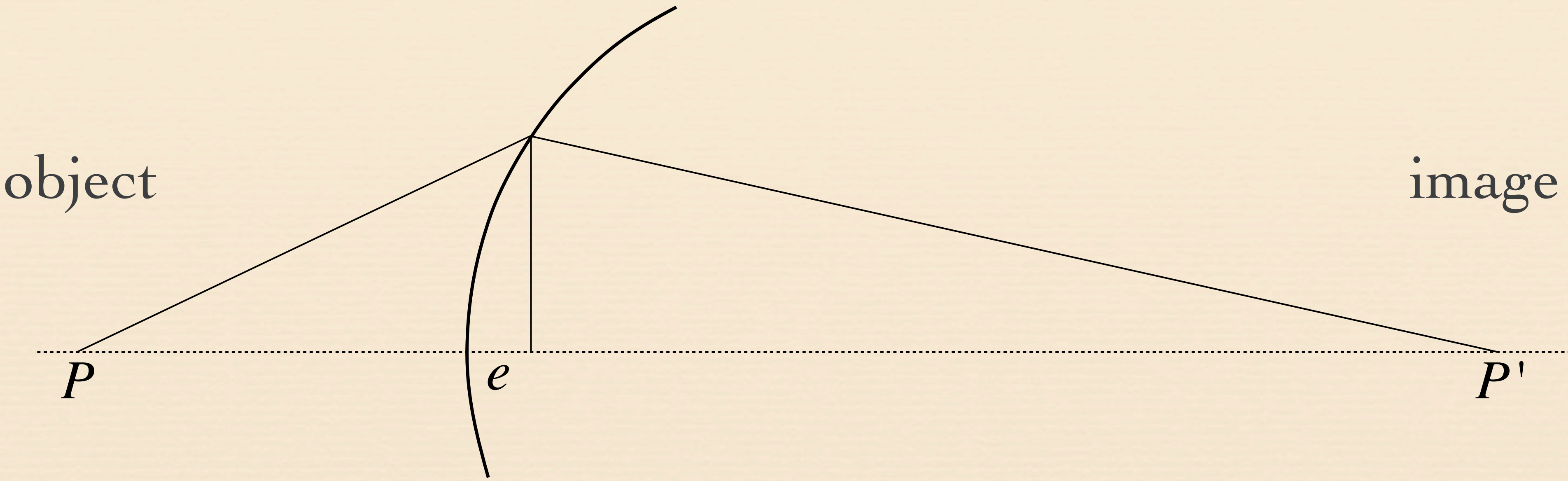
As I mentioned in class, a spherically aberrant image can be thought of as a sharp image (formed by the central rays) + a hazy image (formed by the marginal rays). The look is quite different than a misfocused image, where nothing is sharp. Some people compare it to photographing through a silk stocking. I've never tried this.



(gtmerideth)

(Canon)

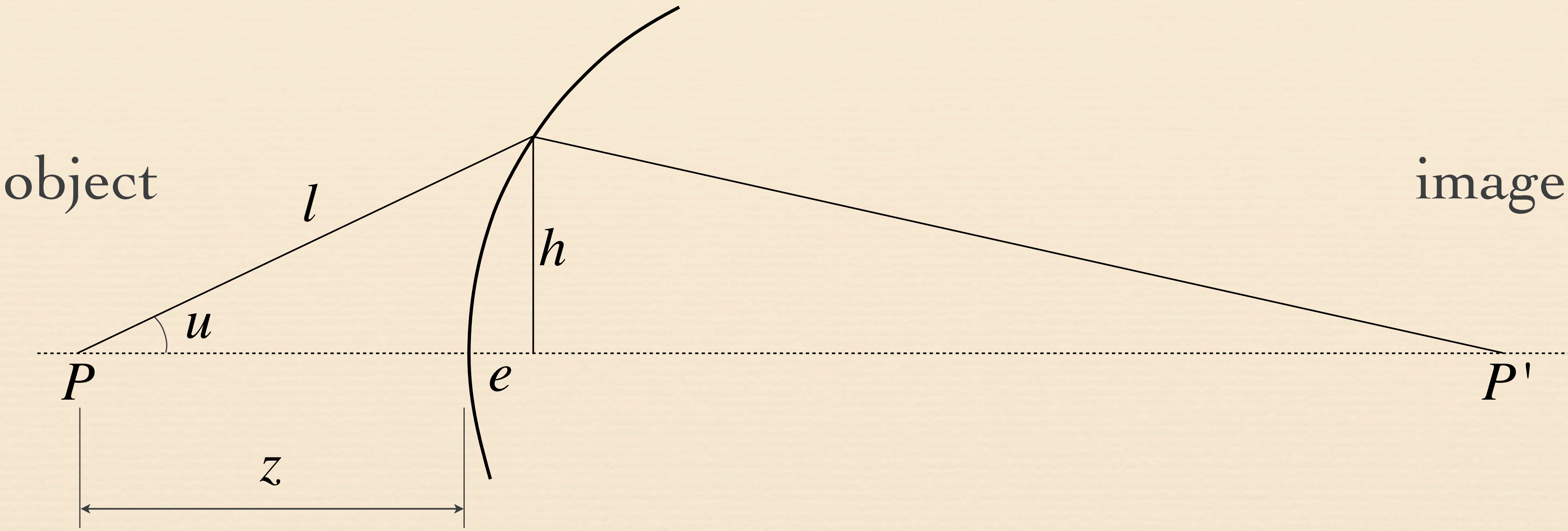
Paraxial approximation



- assume $e \approx 0$

Not responsible on exams
for orange-tinted slides

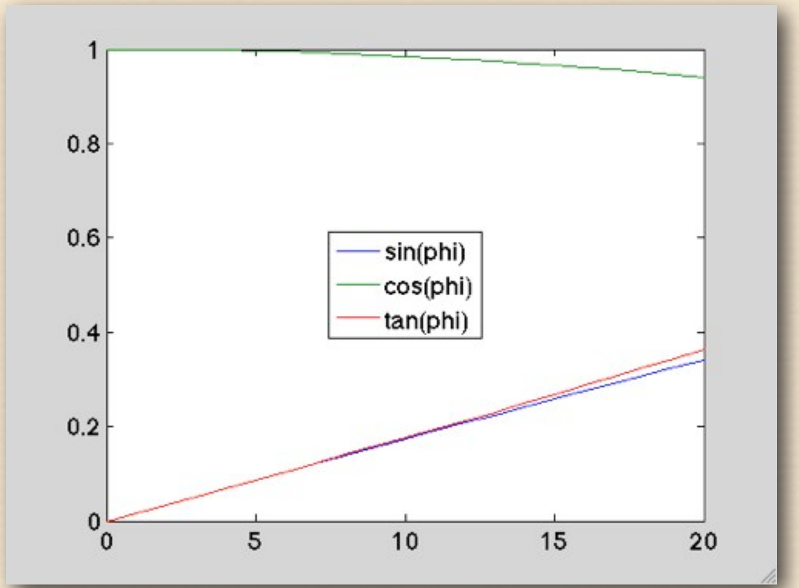
Paraxial approximation



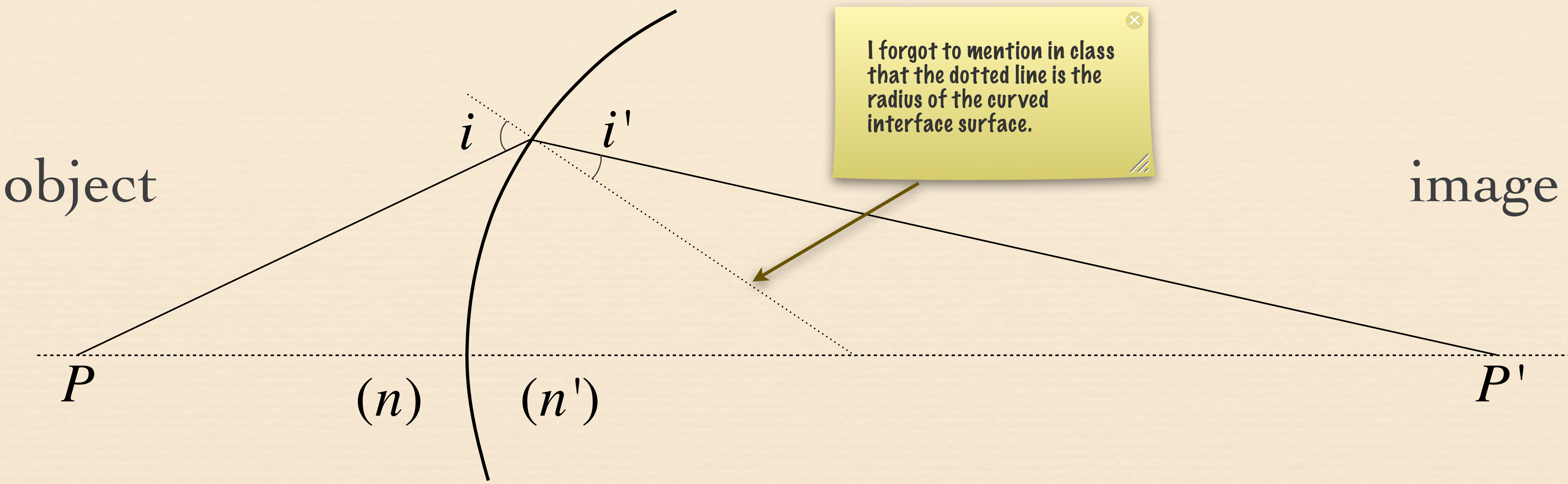
- assume $e \approx 0$
- assume $\sin u = h / l \approx u$ (for u in radians)
- assume $\cos u \approx z / l \approx 1$
- assume $\tan u \approx \sin u \approx u$

The paraxial approximation is a.k.a. first-order optics

- assume first term of $\sin\phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \frac{\phi^7}{7!} + \ldots$
  - i.e. $\sin\phi \approx \phi$
- assume first term of $\cos\phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \frac{\phi^6}{6!} + \ldots$
  - i.e. $\cos\phi \approx 1$
  - so $\tan\phi \approx \sin\phi \approx \phi$



Paraxial focusing



Snell's law:

$$n \sin i \;=\; n' \sin i'$$

paraxial approximation:

$$n\, i \;\approx\; n'\, i'$$

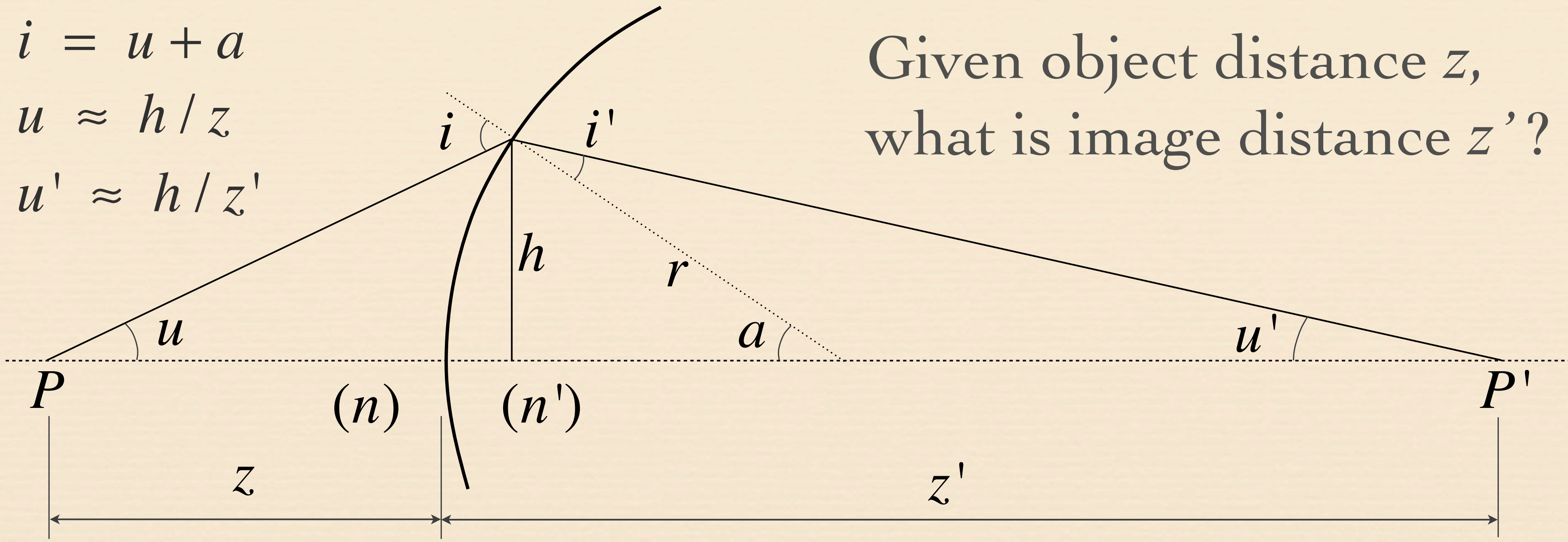
Paraxial focusing

$i = u + a$

$u \approx h / z$

$u' \approx h / z'$

Given object distance z, what is image distance z'?



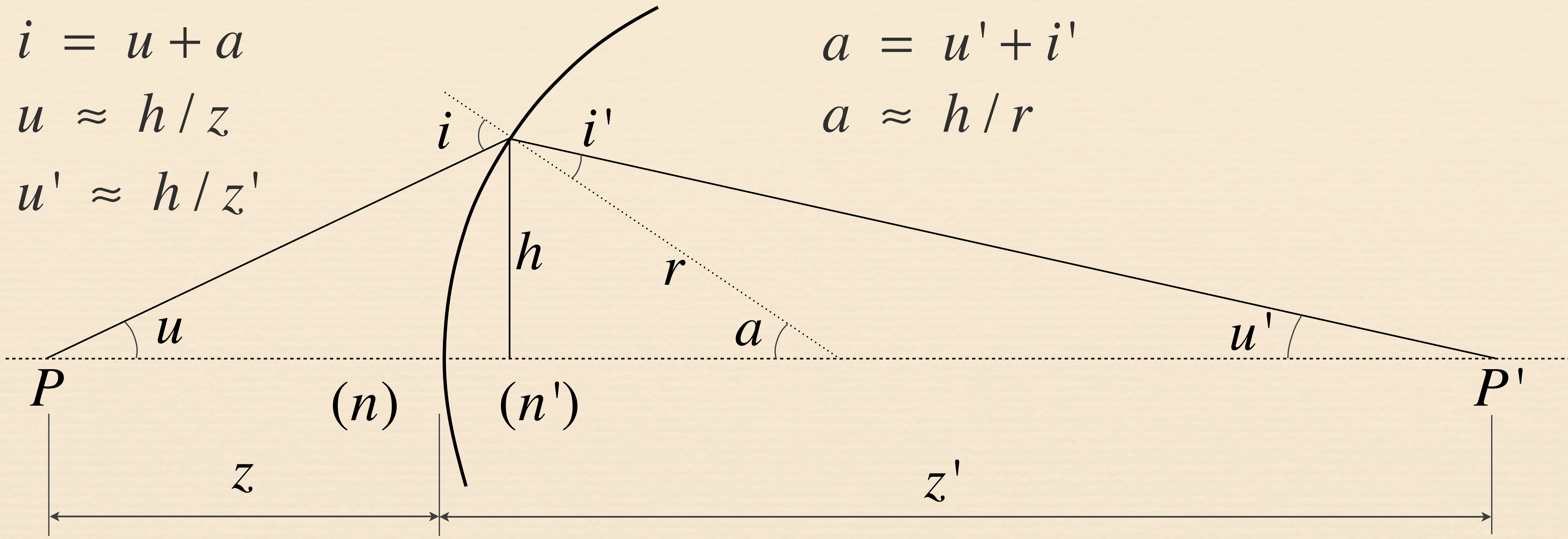
$n\,i \approx n'\,i'$

Paraxial focusing

$i = u + a$

$u \approx h / z$

$u' \approx h / z'$



$a = u' + i'$

$a \approx h / r$

$n\,i \approx n'\,i'$

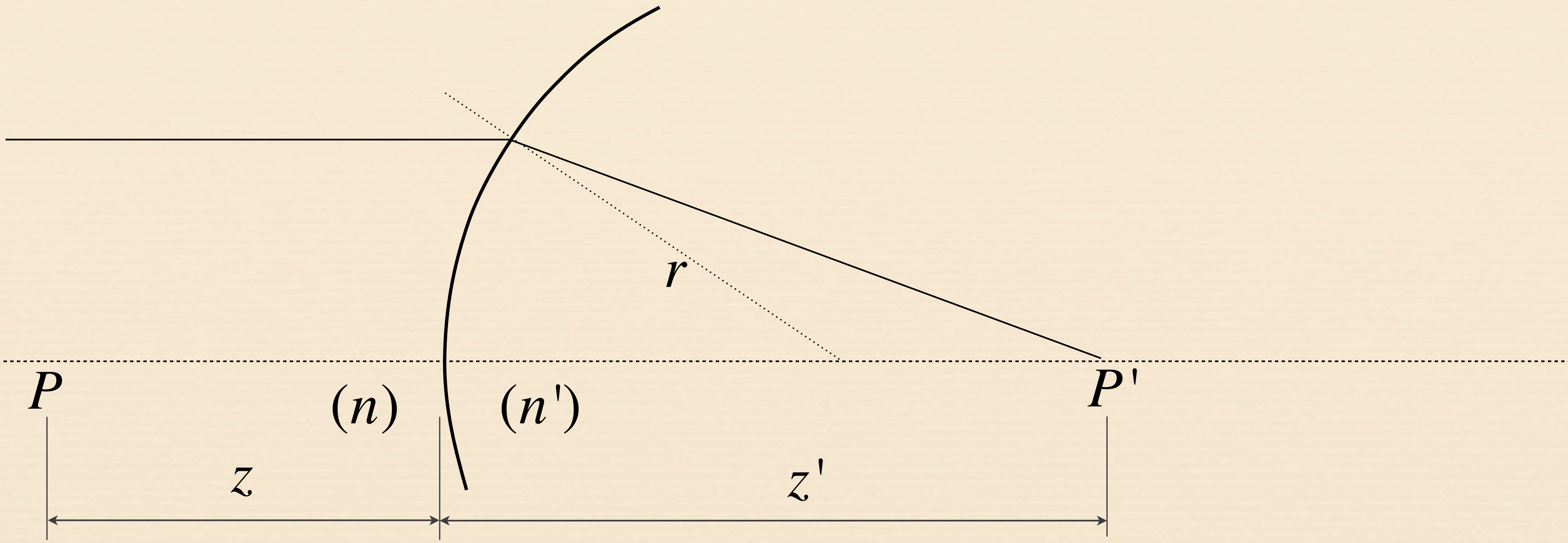
$n\,(u + a) \approx n'\,(a - u')$

$n\,(h / z + h / r) \approx n'\,(h / r - h / z')$

$n / z + n / r \approx n' / r - n' / z'$

- *h* has canceled out, so any ray from *P* will focus to *P'*

Focal length



What happens if z is ∞ ?

$$n / z + n / r \approx n' / r - n' / z'$$

$$n / r \approx n' / r - n' / z'$$

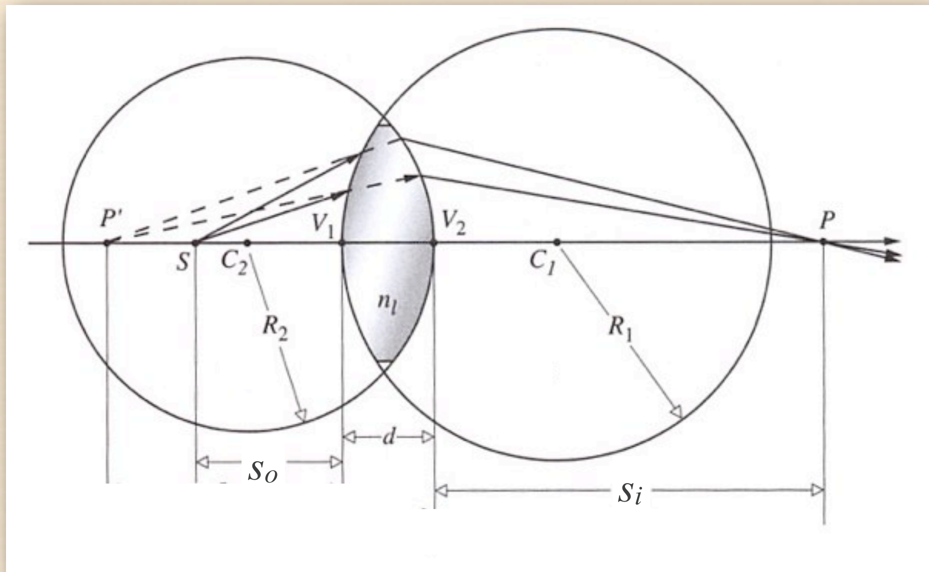
$$z' \approx (r\, n') / (n' - n)$$

Here's an example: if n = 1, n′ = 1.5, and r = 20mm, then z′ = f = 60mm.

✦ $f \triangleq$ focal length $= z'$

Lensmaker's formula

- using similar derivations, one can extend these results to two spherical interfaces forming a lens in air



(Hecht, edited)

- as *d → 0 (thin lens approximation)*, we obtain the lensmaker's formula

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Gaussian lens formula

- Starting from the lensmaker's formula

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \qquad \text{(Hecht, eqn 5.15)}$$

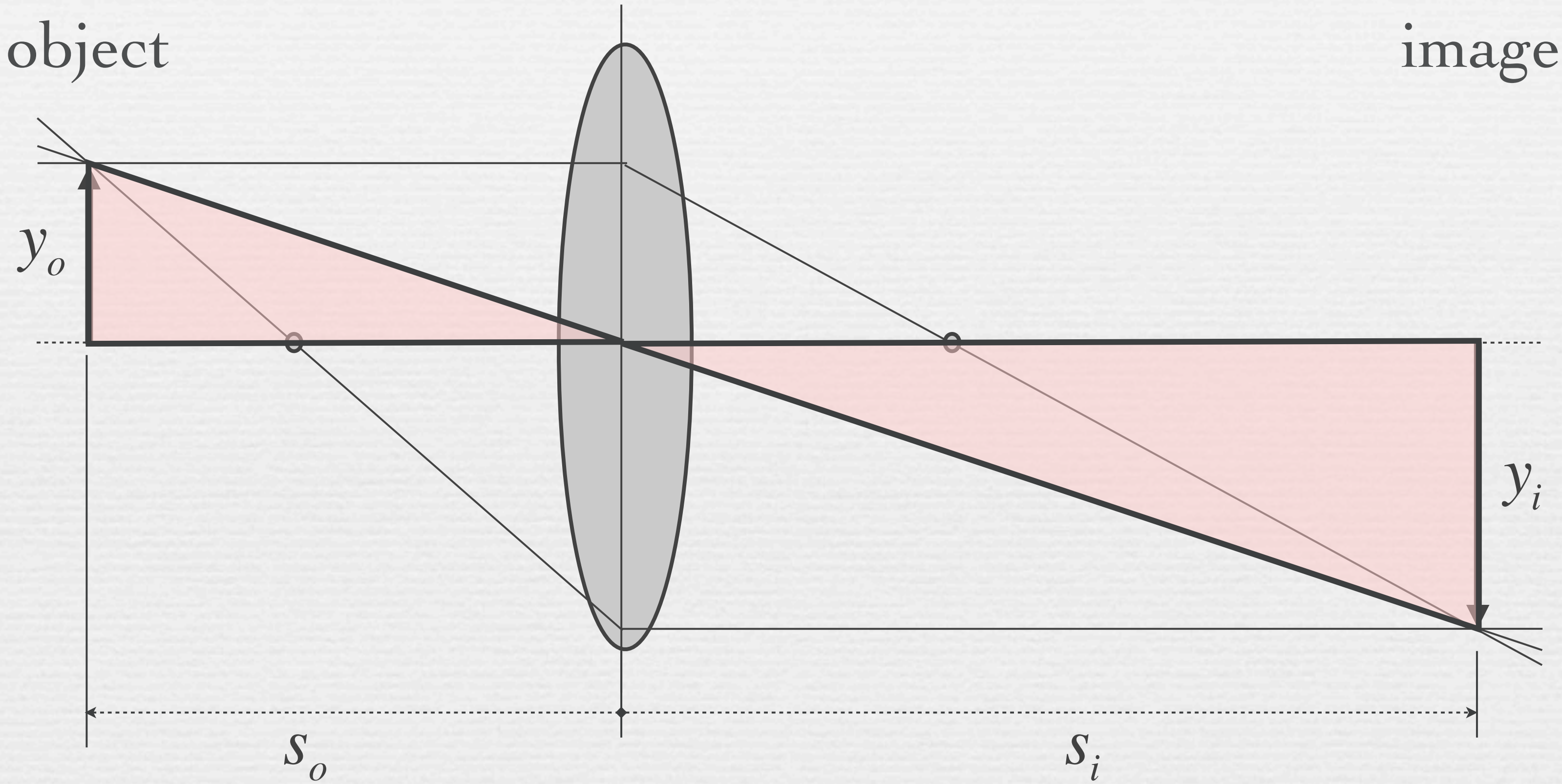
- and recalling that as object distance s_o is moved to infinity, image distance s_i becomes focal length f_i , we get

$$\frac{1}{f_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \qquad \text{(Hecht, eqn 5.16)}$$

- Equating these two, we get the Gaussian lens formula

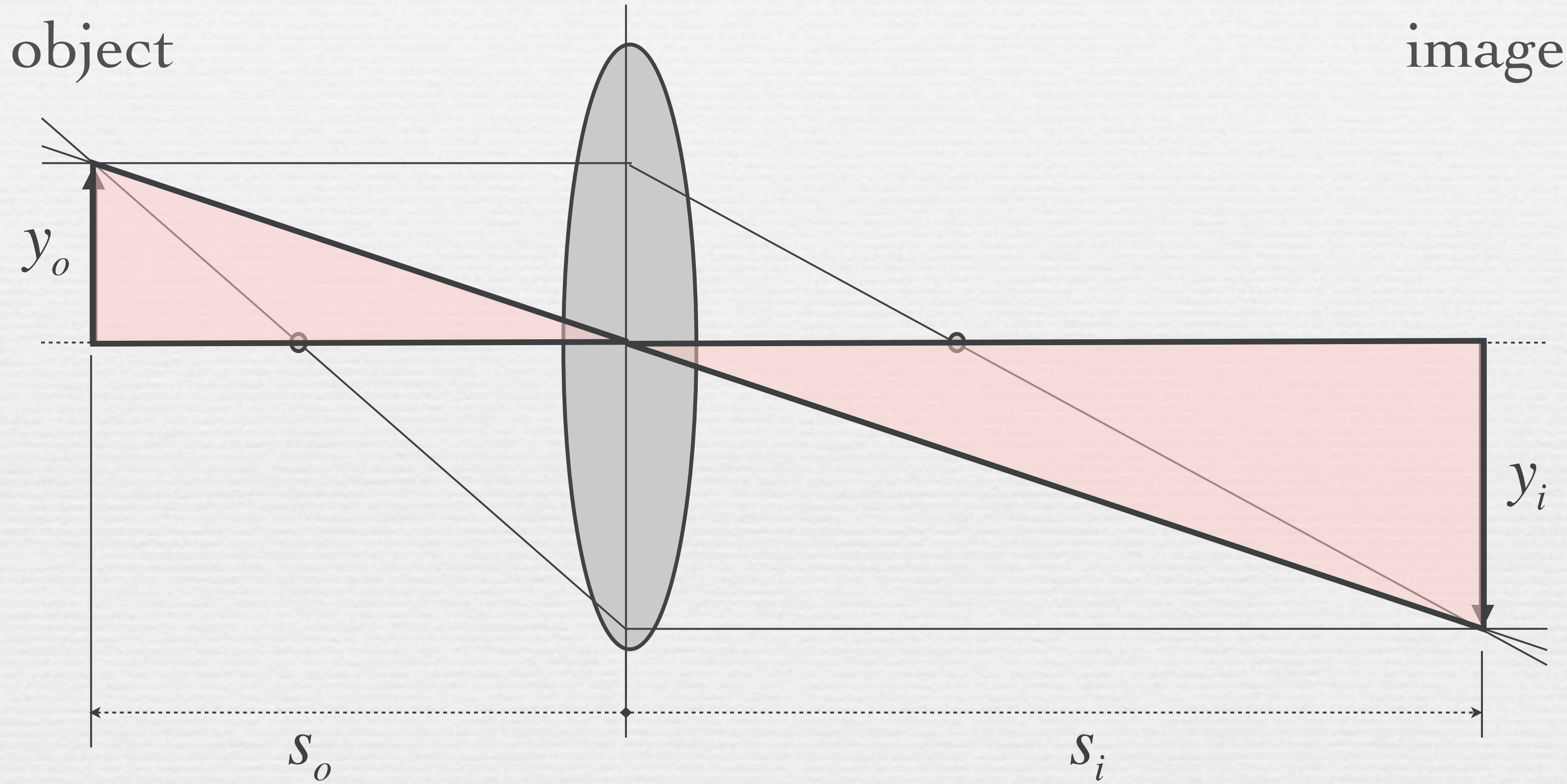
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f_i}. \qquad \text{(Hecht, eqn 5.17)}$$

From Gauss's ray construction to the Gaussian lens formula



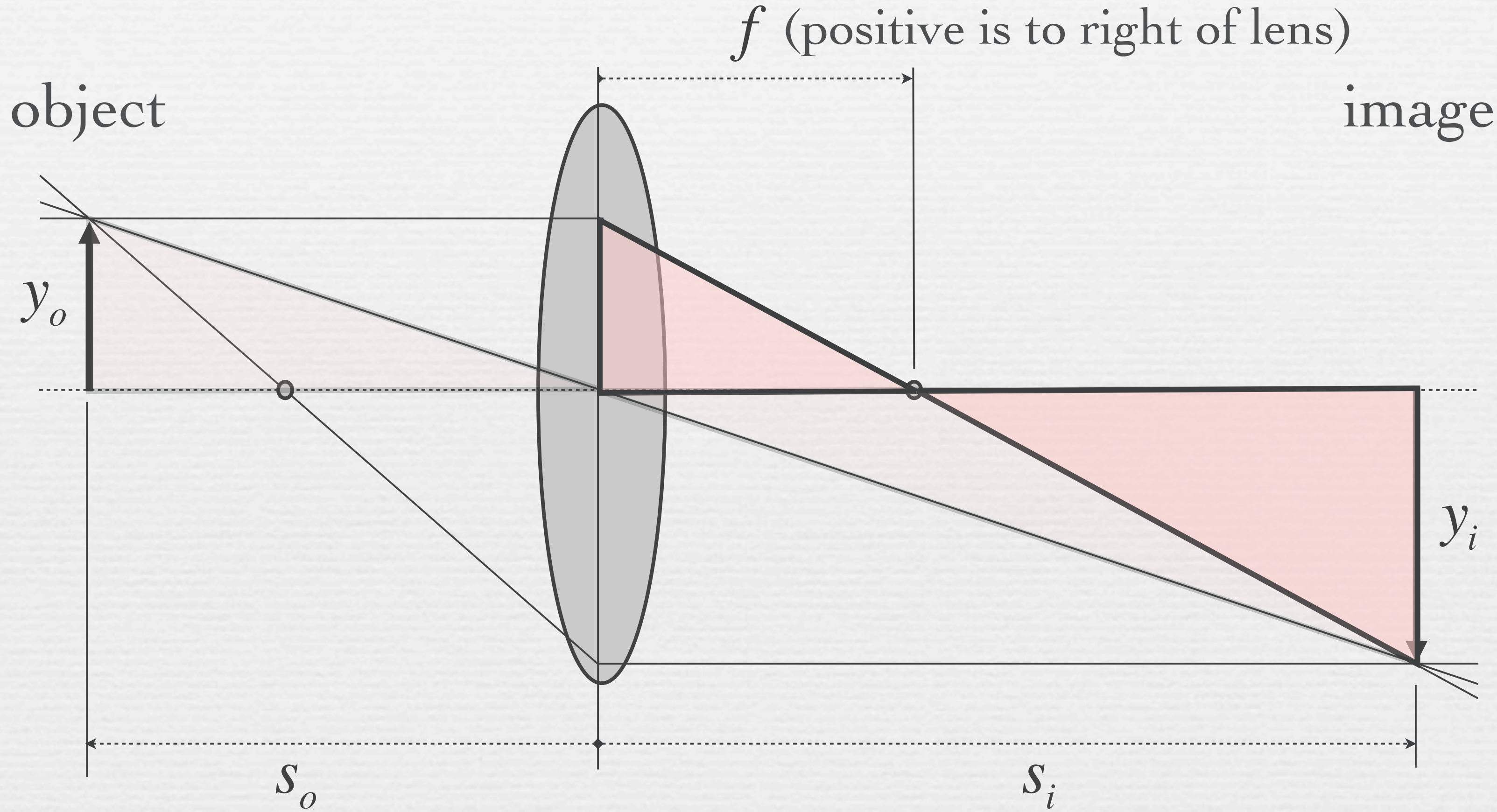
- positive s_i is rightward, positive s_o is leftward
- positive y is upward

From Gauss's ray construction to the Gaussian lens formula



$$\frac{|y_i|}{y_o} = \frac{s_i}{s_o}$$

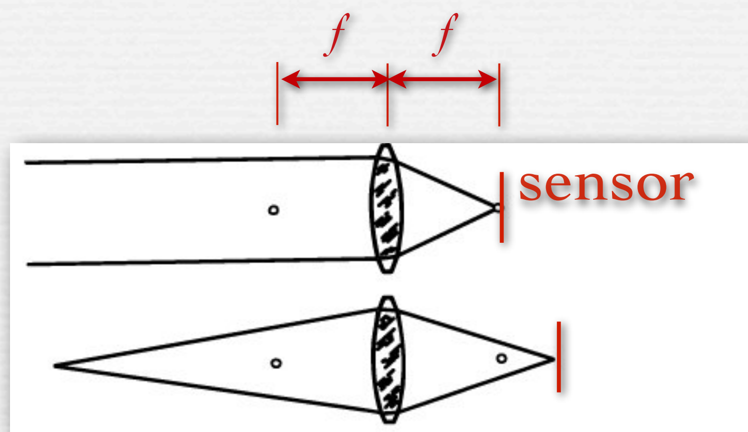
From Gauss's ray construction to the Gaussian lens formula



$$\frac{|y_i|}{y_o} = \frac{s_i}{s_o} \quad \text{and} \quad \frac{|y_i|}{y_o} = \frac{s_i - f}{f} \quad \ldots\ldots \quad \boxed{\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}}$$

Changing the focus distance

- to focus on objects at different distances, move sensor relative to lens



To help reduce confusion between sensor-lens distance s_i, which represents focusing a camera, and focal length f, which represents zooming a camera, we've added sensor size and field of view (FOV) to the applet I showed in class on 4/6/10. Try it out!

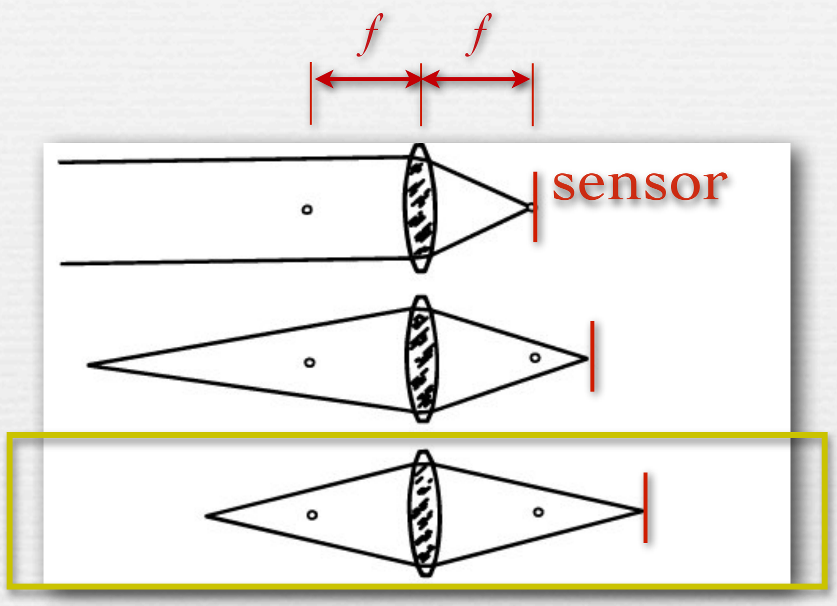
(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/gaussian.html

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Changing the focus distance

- to focus on objects at different distances, move sensor relative to lens



- at $s_o = s_i = 2f$ we have 1:1 imaging, because

$$\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}$$

In 1:1 imaging, if the sensor is 36mm wide, an object 36mm wide will fill the frame.

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

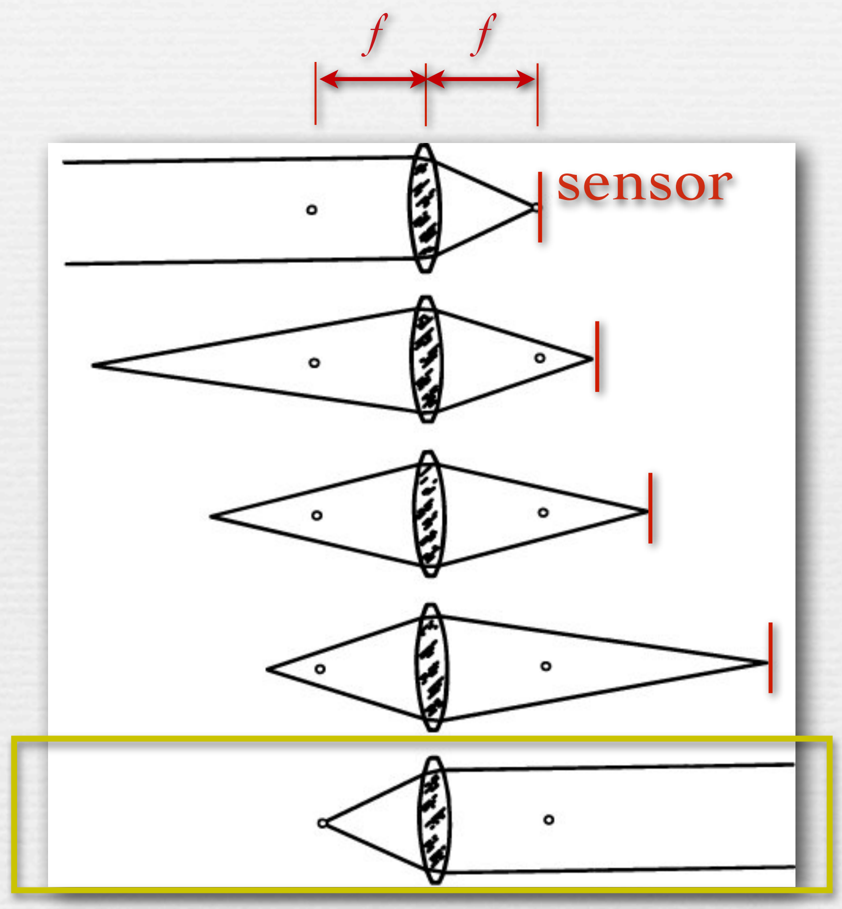
Changing the focus distance

- to focus on objects at different distances, move sensor relative to lens

- at $s_o = s_i = 2f$ we have 1:1 imaging, because

$$\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}$$

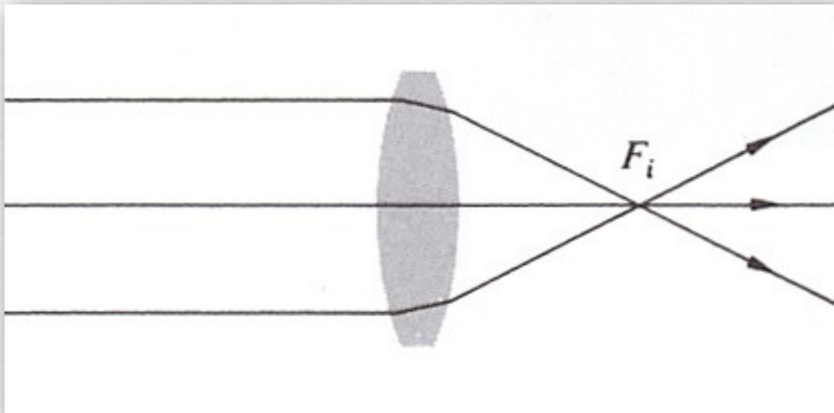
- can't focus on objects closer to lens than its focal length f



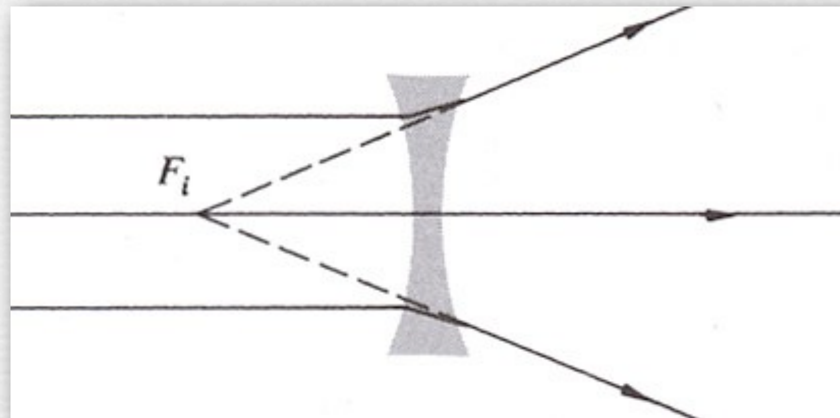
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Convex versus concave lenses

(Hecht)



rays from a convex lens converge

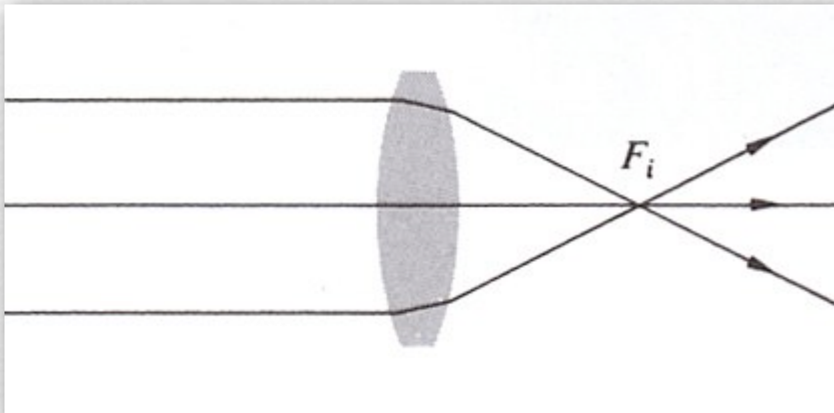


rays from a concave lens diverge

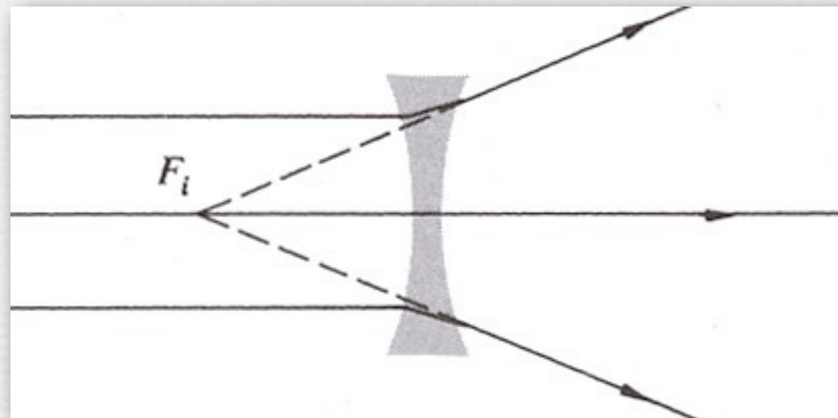
- positive focal length *f* means parallel rays from the left converge to a point on the right
- negative focal length *f* means parallel rays from the left converge to a point on the left (dashed lines above)

Convex versus concave lenses

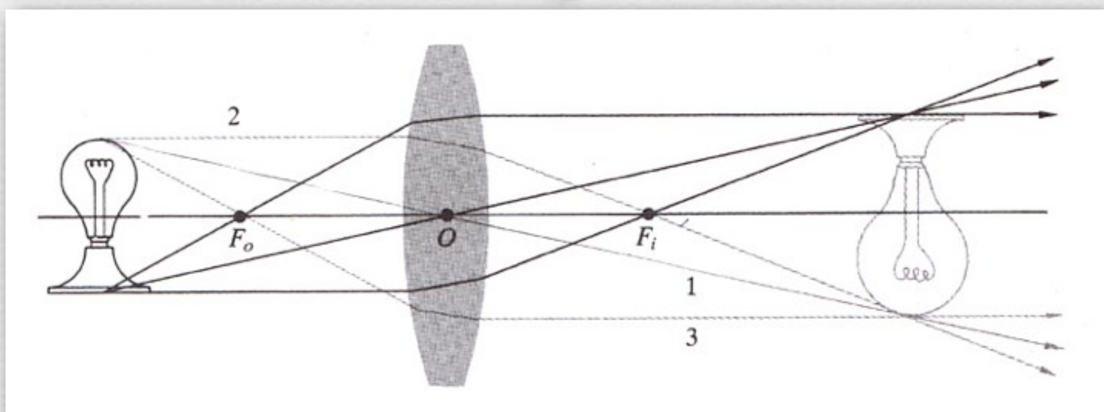
(Hecht)



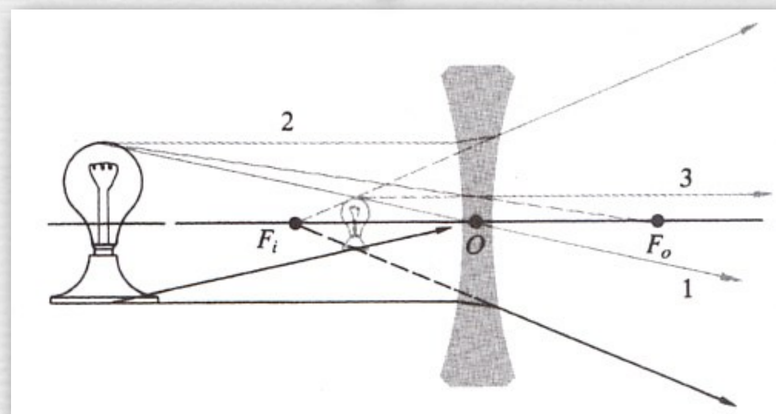
rays from a convex lens converge



rays from a concave lens diverge

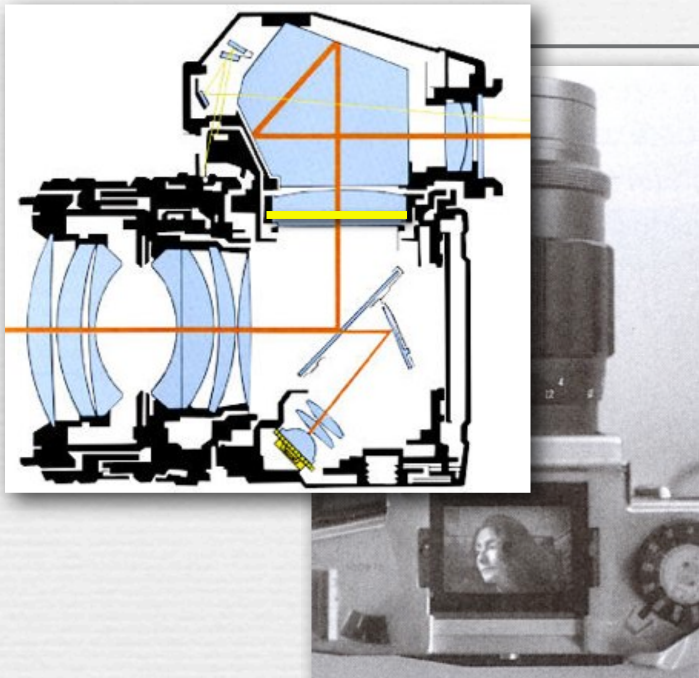


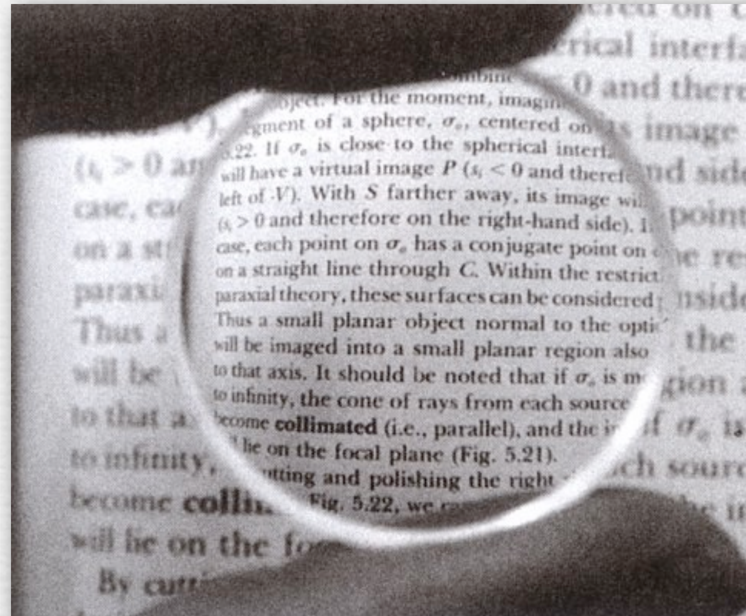
...producing a real image

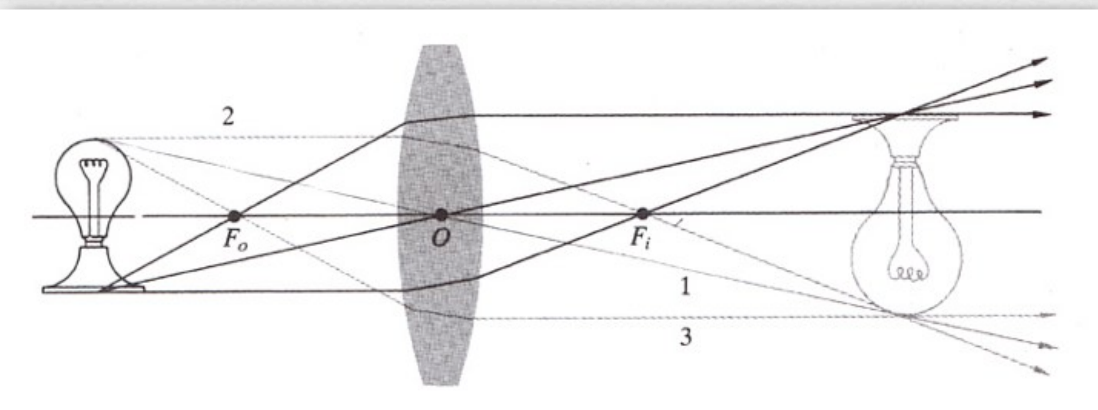


...producing a virtual image

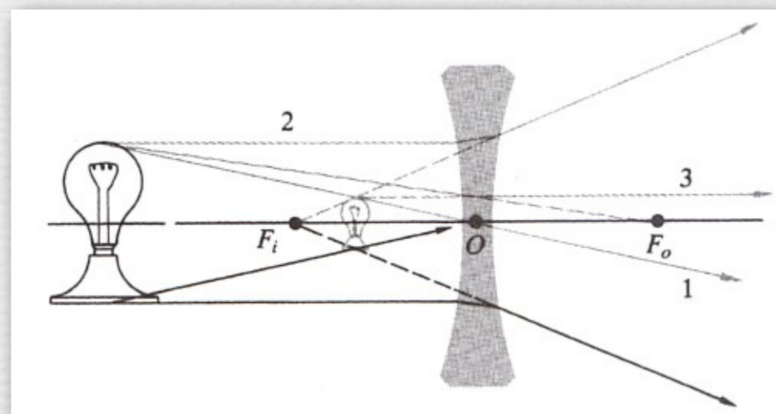
Convex versus concave lenses







...producing a real image



...producing a virtual image

The power of a lens

$$P = \frac{1}{f}$$

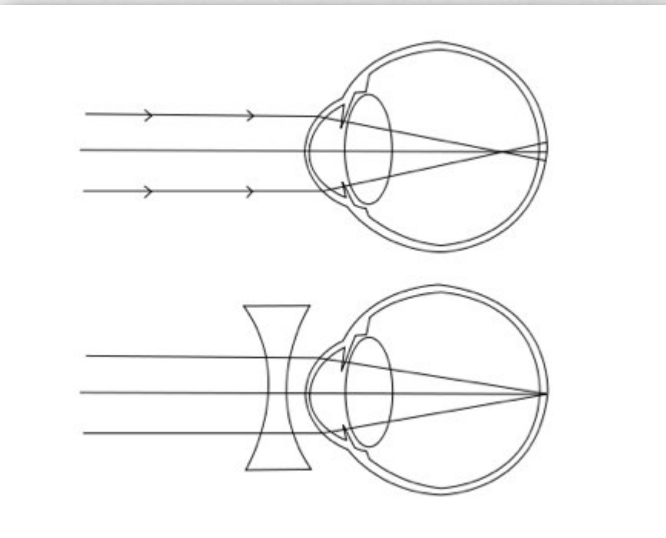
- units are meters^{-1}
- a.k.a. diopters

In class some people guessed nearsightedness as my problem. Indeed that word is synonymous with myopia. The opposite is hyperopia, which is synonymous with farsightedness.

What this means in practice is that I have trouble focusing on objects that are at infinity, as the first drawing below shows. However, I have no trouble focusing on nearby objects. Looking at the drawing, you can imagine that if the object were closer to my eye, then the image distance would increase (according to Gauss's lens formula), and the object would then come to a focus on my retina. This would happen without corrective eyeglasses. This is why you see me remove my eyeglasses when I need to look at my laptop screen.

- my eyeglasses have the prescription
  - right eye: -0.75 diopters
  - left eye: -1.00 diopters

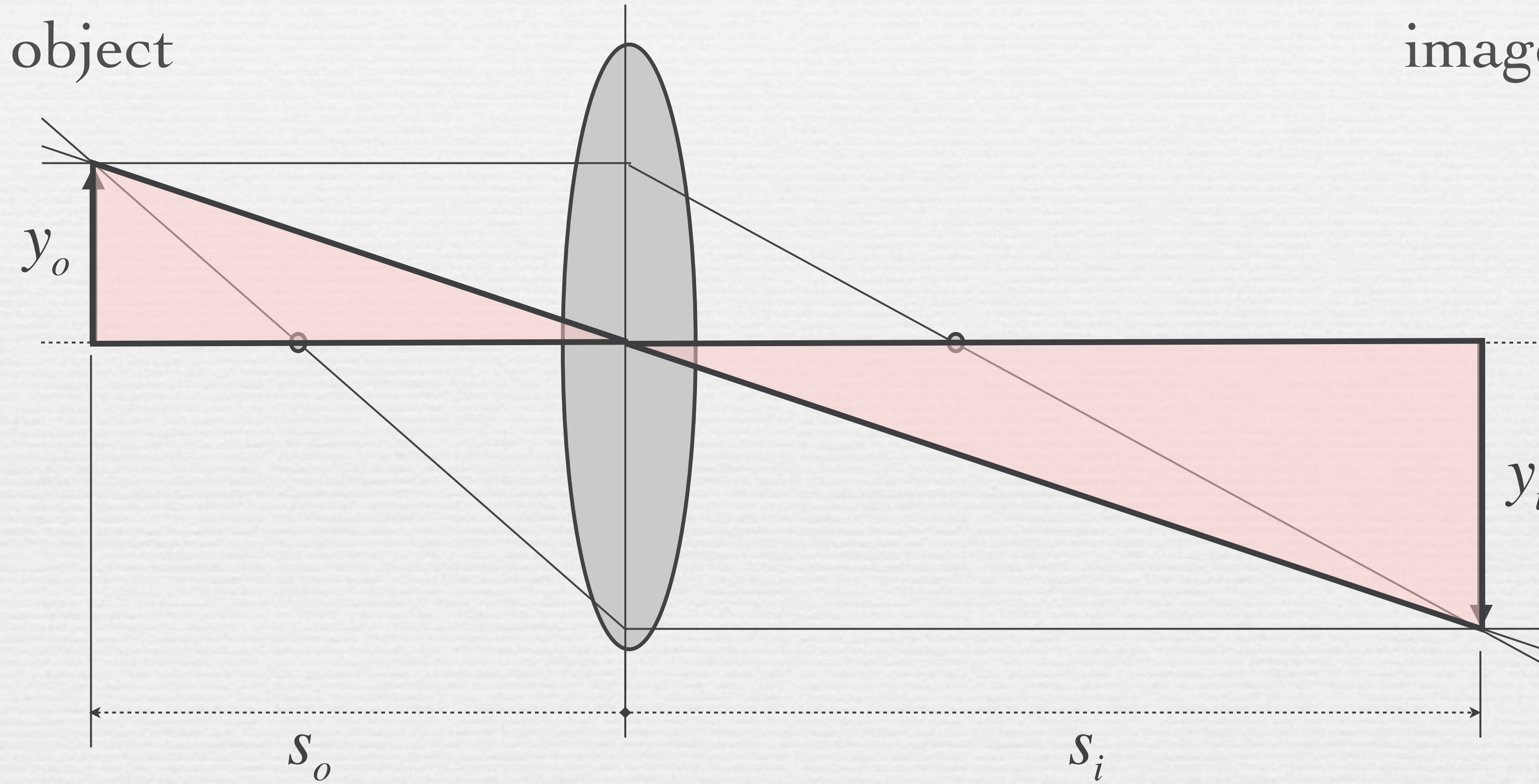
(wikipedia)



Q. What's wrong with me?

A. Myopia (nearsightedness)

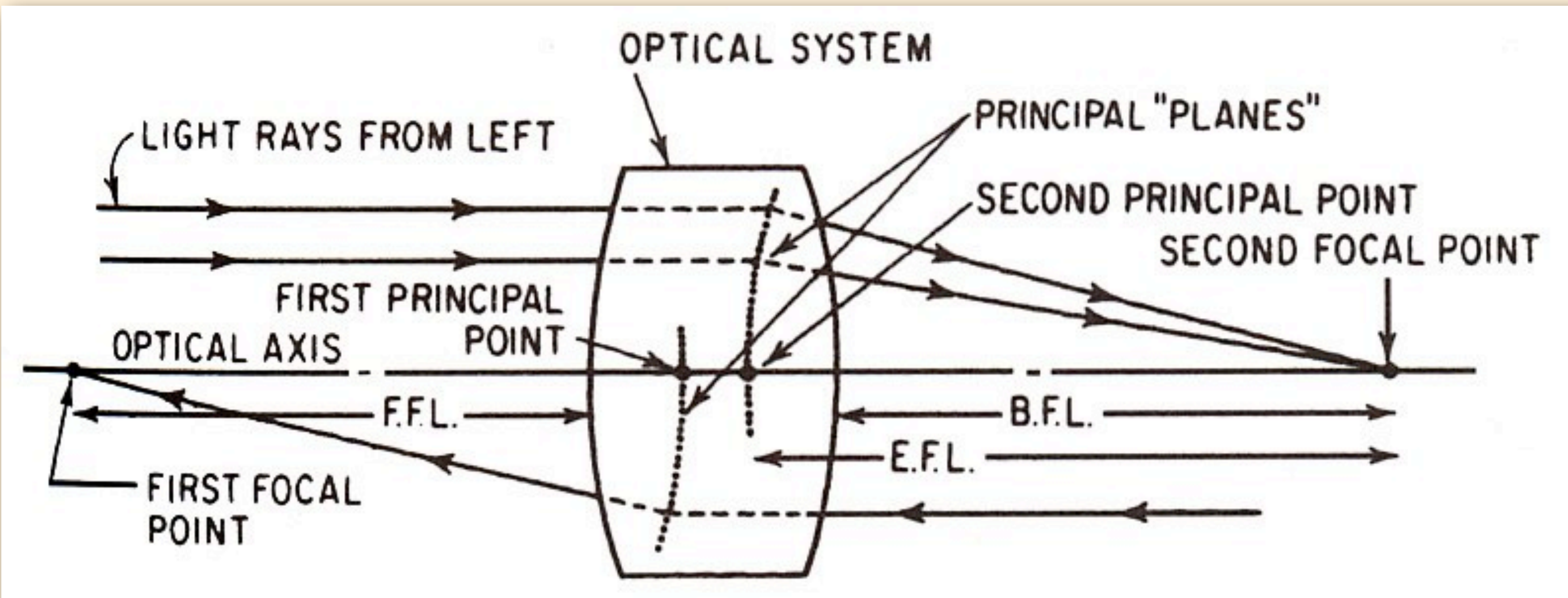
Magnification



$$M_T \triangleq \frac{y_i}{y_o} = -\frac{s_i}{s_o}$$

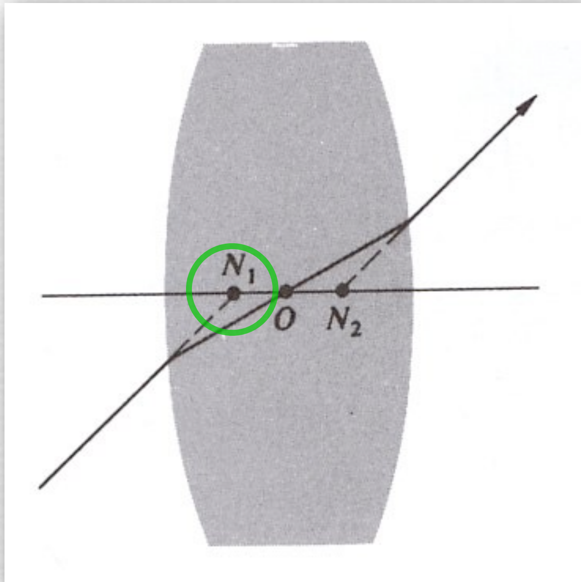
Thick lenses

- an optical system may contain many lenses, but can be characterized by a few numbers



(Smith)

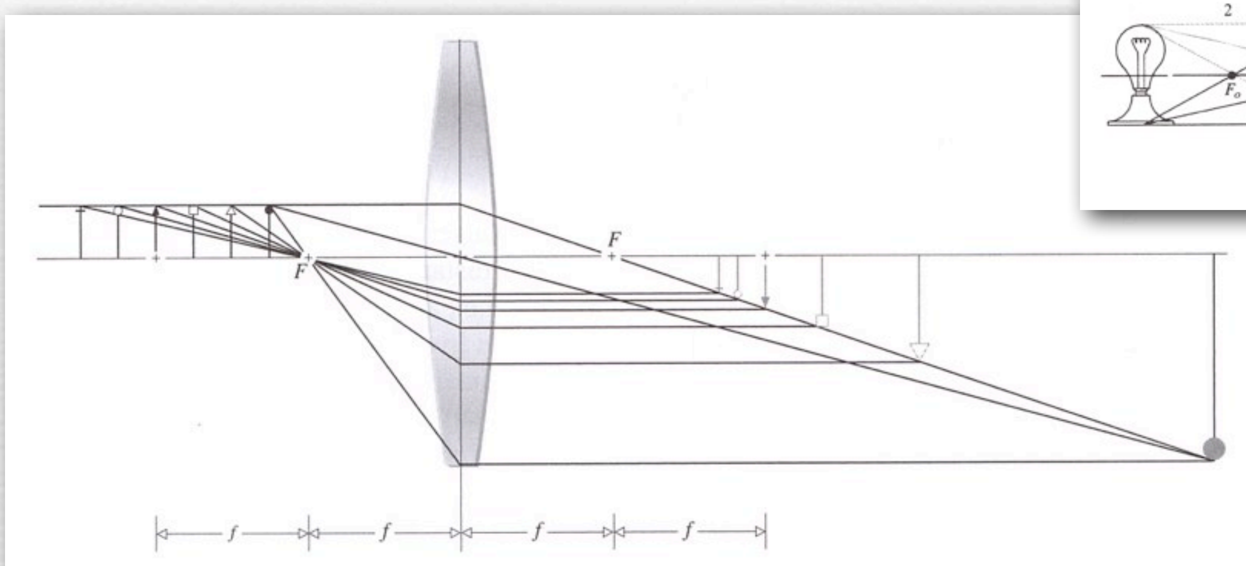
Center of perspective

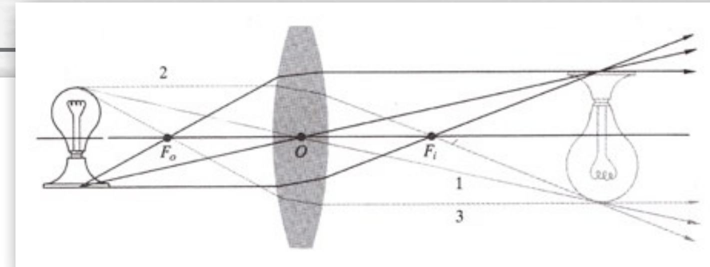


(Hecht)

- in a thin lens, the *chief ray* traverses the lens (through its optical center) without changing direction
- in a thick lens, the intersections of this ray with the optical axis are called the *nodal points*
- for a lens in air, these coincide with the *principal points*
- the first nodal point is the *center of perspective*

Lenses perform a 3D perspective transform





(Hecht)

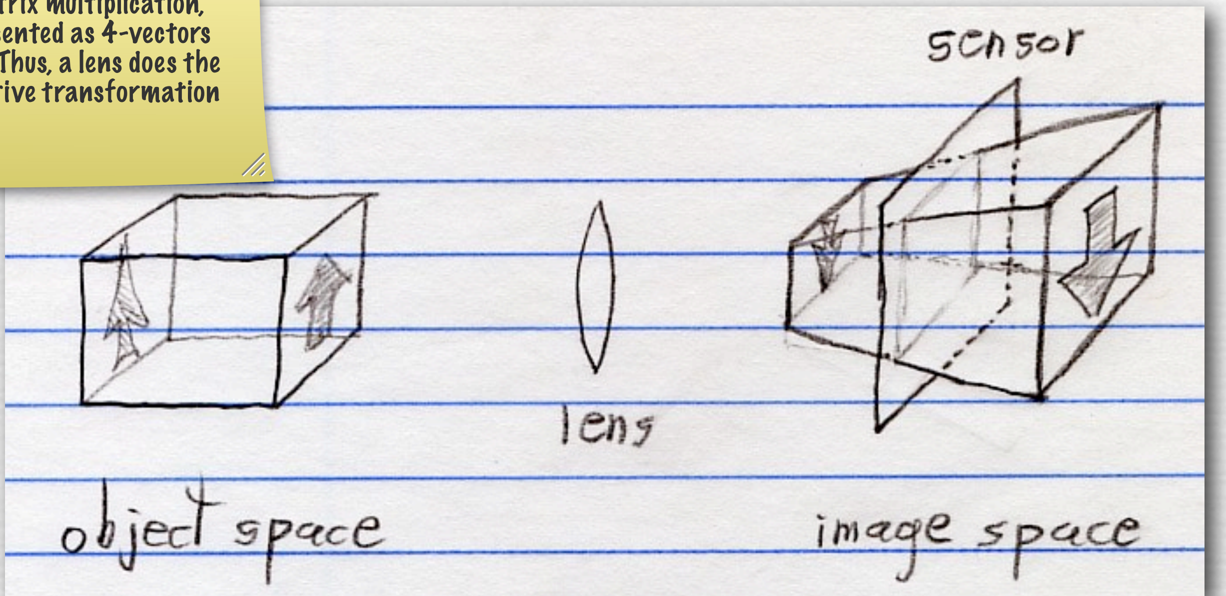
(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/thinlens.html

- lenses transform a 3D object to a 3D image; the sensor extracts a 2D slice from that image
- as an object moves linearly (in Z), its image moves non-proportionately (in Z)
- as you move a lens linearly relative to the sensor, the in-focus object plane moves non-proportionately
- as you refocus a camera, the image changes size !

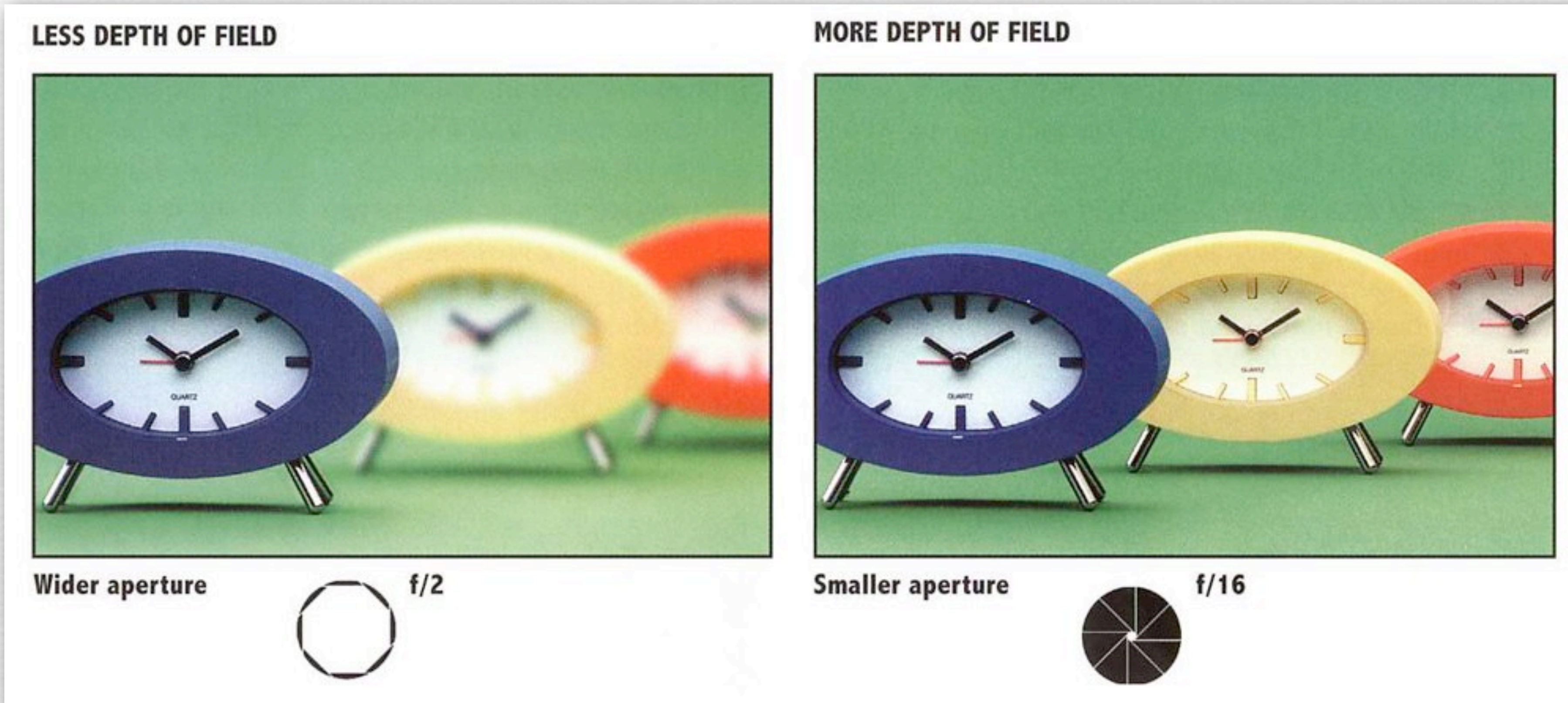
Lenses perform a 3D perspective transform (contents of whiteboard)

If you've taken a computer graphics course, you'll know that this 3D perspective transform is implemented using a 4 x 4 matrix multiplication, where object points are represented as 4-vectors in homogeneous coordinates. Thus, a lens does the same thing as a 4 x 4 perspective transformation in computer graphics.



- a cube in object space is transformed by a lens into a 3D frustum in image space, with the orientations shown by the arrows
- in computer graphics this transformation is modeled as a 4 × 4 matrix multiplication of 3D points expressed in 4D homogenous coordinates
- in photography a sensor extracts a 2D slice from the 3D frustum; on this slice some objects may be sharply focused; others may be blurry

Depth of field

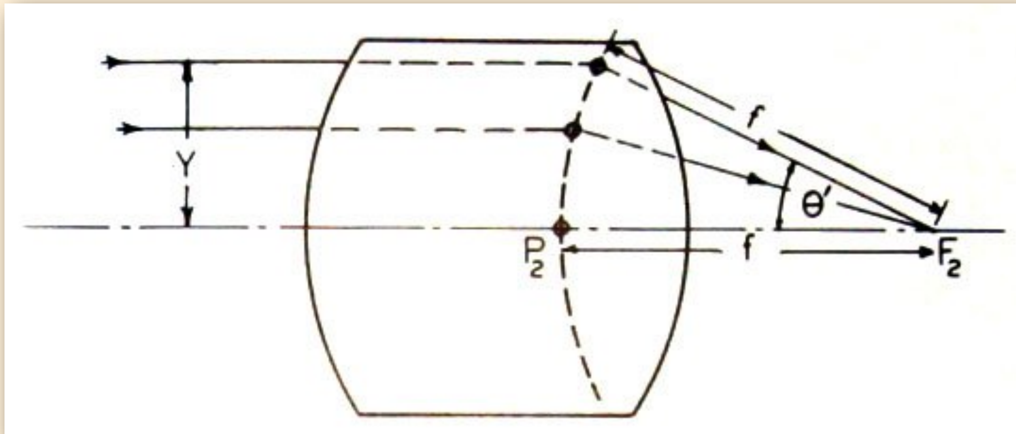


(London)

$$N = \frac{f}{A}$$

- lower N means a wider aperture and less depth of field

How low can N be?



(Kingslake)

- principal planes are the paraxial approximation of a spherical "equivalent refracting surface"

$$N = \frac{1}{2\sin\theta'}$$

Canon EOS 50mm f/1.0 (discontinued)

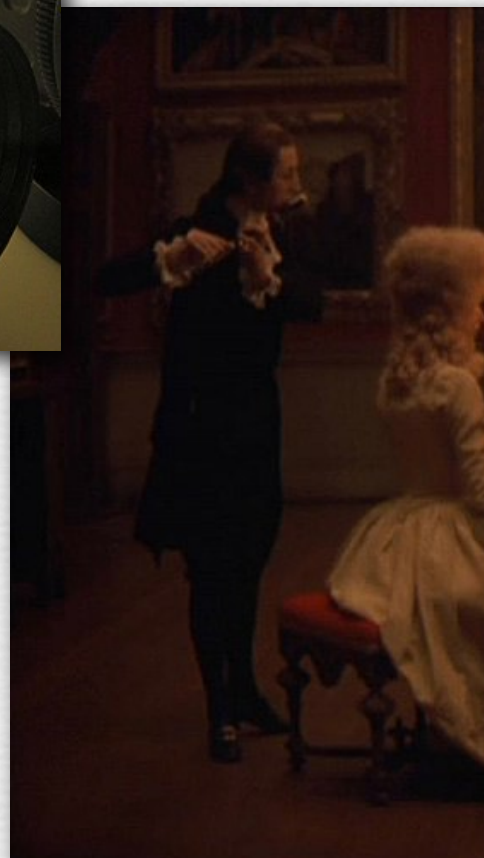
- lowest possible N in air is f/0.5
- lowest N in SLR lenses is f/1.0

Cinematography by candlelight

Stanley Kubrick,
Barry Lyndon,
1975

- Zeiss 50mm f/0.7 Planar lens
  - originally developed for NASA's Apollo missions
  - very shallow depth of field in closeups (small object distance)

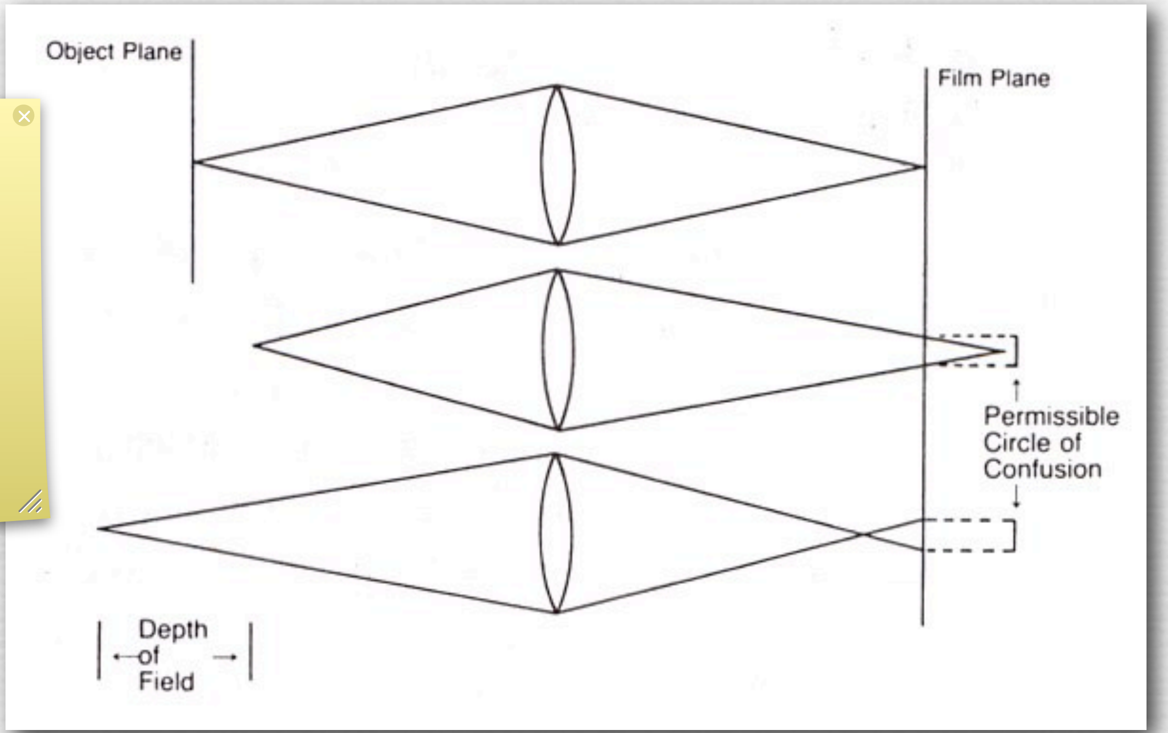
Cinematography by candlelight



Stanley Kubrick,
Barry Lyndon,
1975

- Zeiss 50mm f/0.7 Planar lens
  - originally developed for NASA's Apollo missions
  - very shallow depth of field in closeups (small object distance)

Circle of confusion (C)

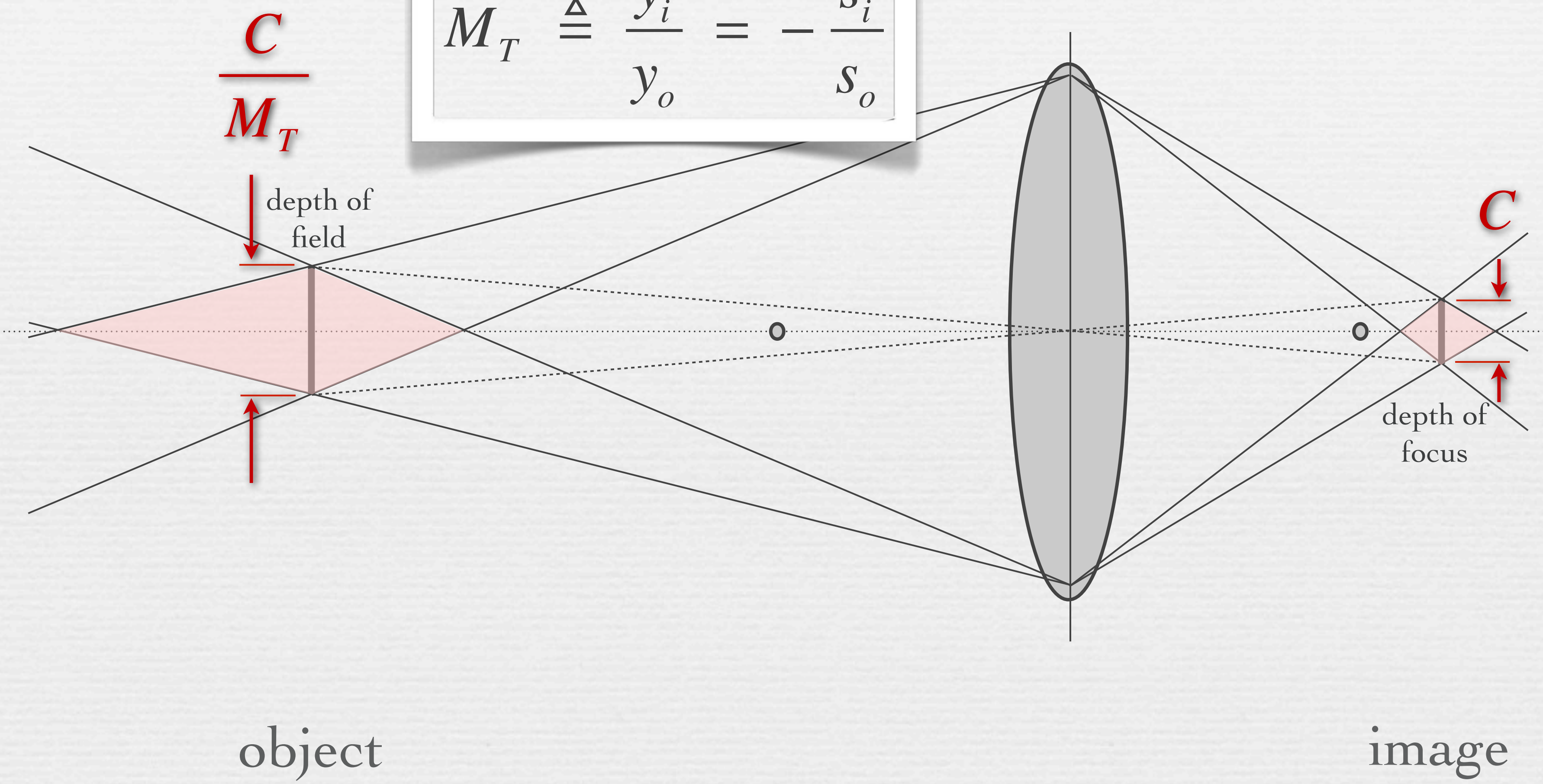


Note added 4/13/10. The right way to factor reproduction medium, viewing distance, and human vision into deciding what circle of confusion is right for a situation is to compute the retinal arc subtended the circle, measured in degrees. I covered this in the Sensing & Pixels lecture on April 13.

- C depends on sensing medium, reproduction medium, viewing distance, human vision,...
  - for print from 35mm film, 0.02mm (on negative) is typical
  - for high-end SLR, 6μ is typical (1 pixel)
  - larger if downsizing for web, or lens is poor

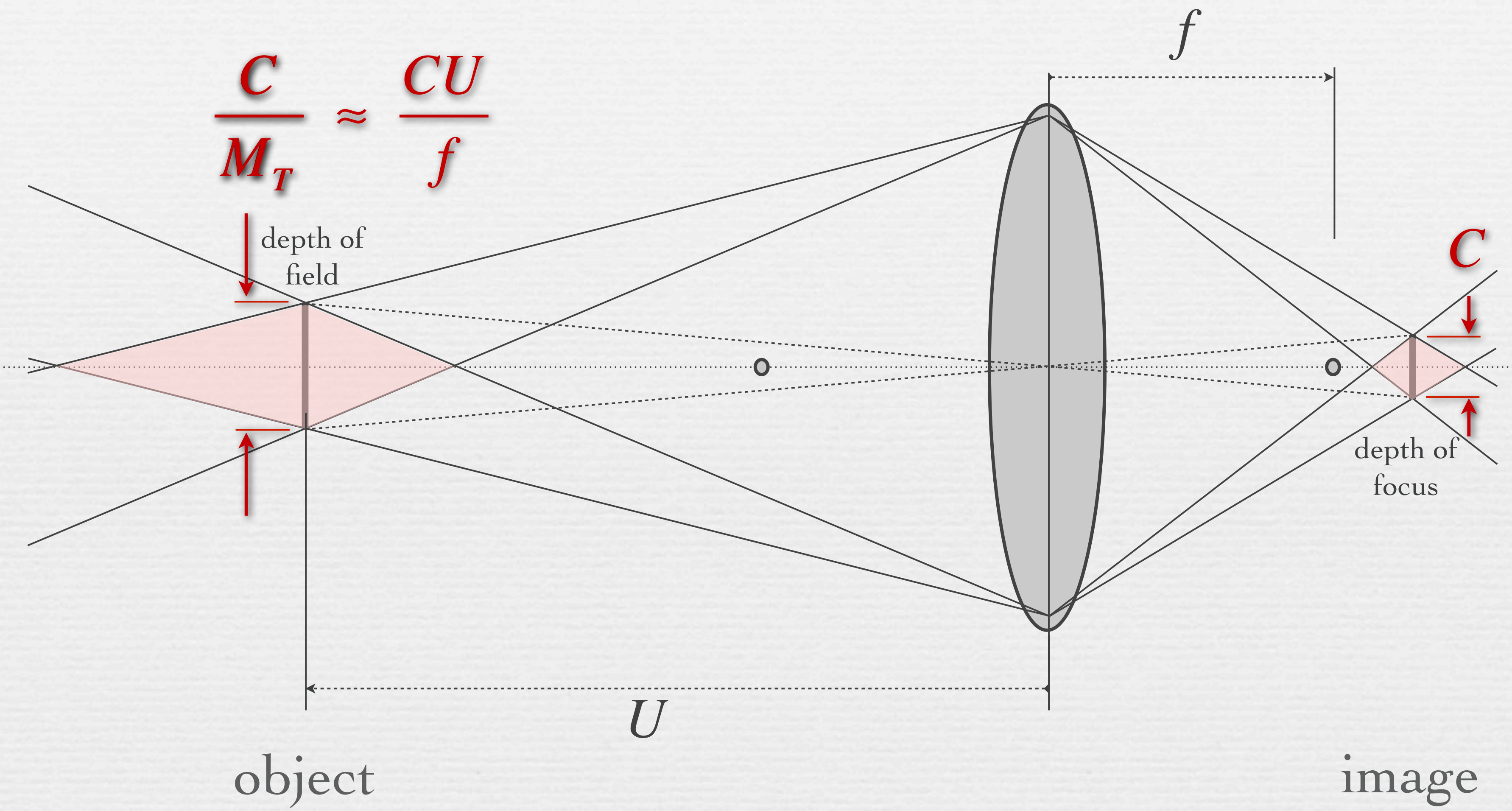
Depth of field formula

$$M_T \triangleq \frac{y_i}{y_o} = -\frac{s_i}{s_o}$$



- DoF is asymmetrical around the in-focus object plane
- conjugate in object space is typically bigger than C

Depth of field formula



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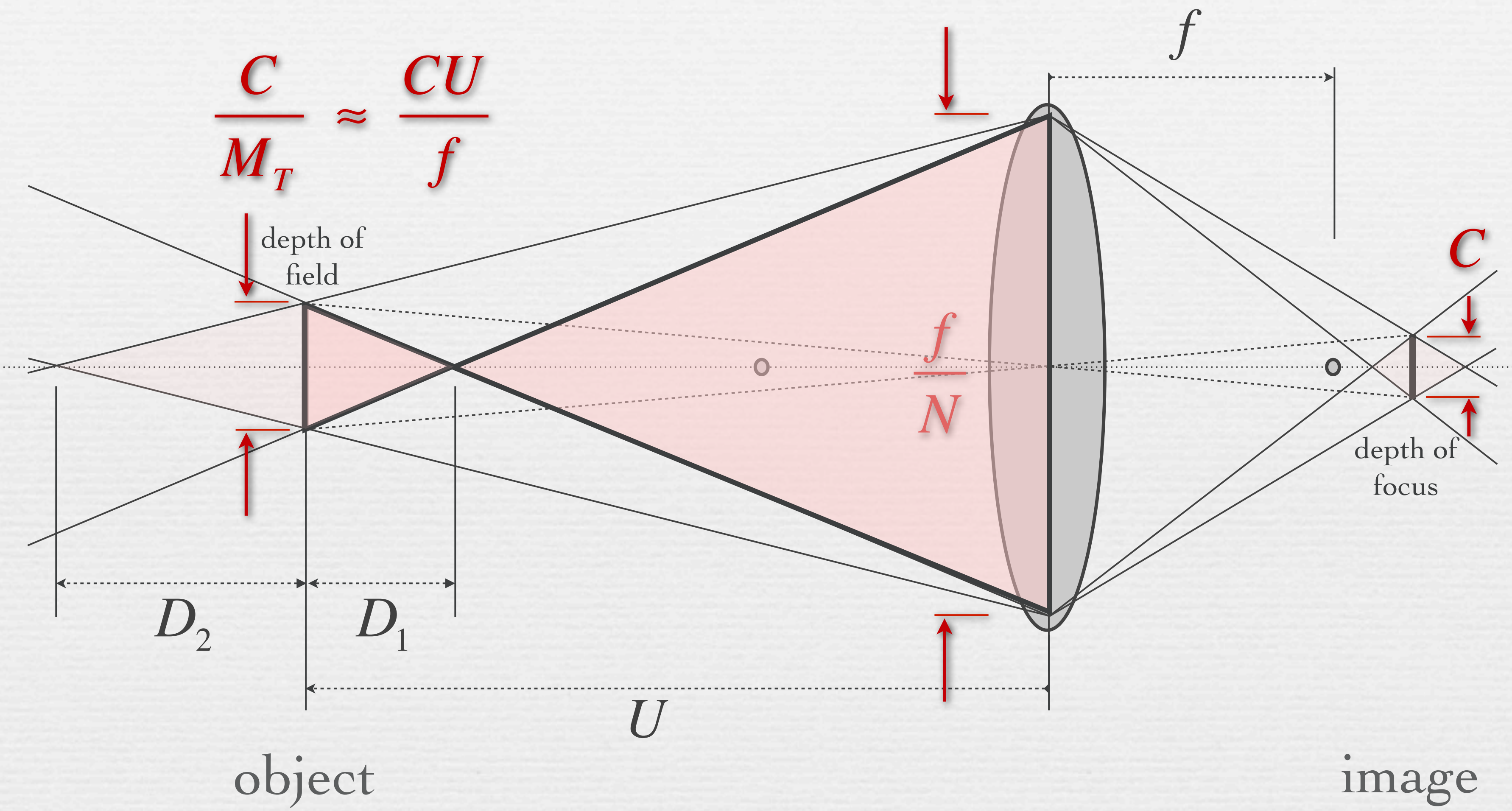
Depth of field formula



$$\frac{D_1 f}{CU} = \frac{U - D_1}{f / N} \quad \ldots\ldots \quad D_1 = \frac{NCU^2}{f^2 + NCU} \qquad D_2 = \frac{NCU^2}{f^2 - NCU}$$

Depth of field formula

$$D_{TOT} = D_1 + D_2 = \frac{2NCU^2 f^2}{f^4 - N^2C^2U^2}$$

- $N^2C^2D^2$ can be ignored when conjugate of circle of confusion is small relative to the aperture

$$\boxed{D_{TOT} \approx \frac{2NCU^2}{f^2}}$$

- where
  - N is F-number of lens
  - C is circle of confusion (on image)
  - U is distance to in-focus plane (in object space)
  - f is focal length of lens

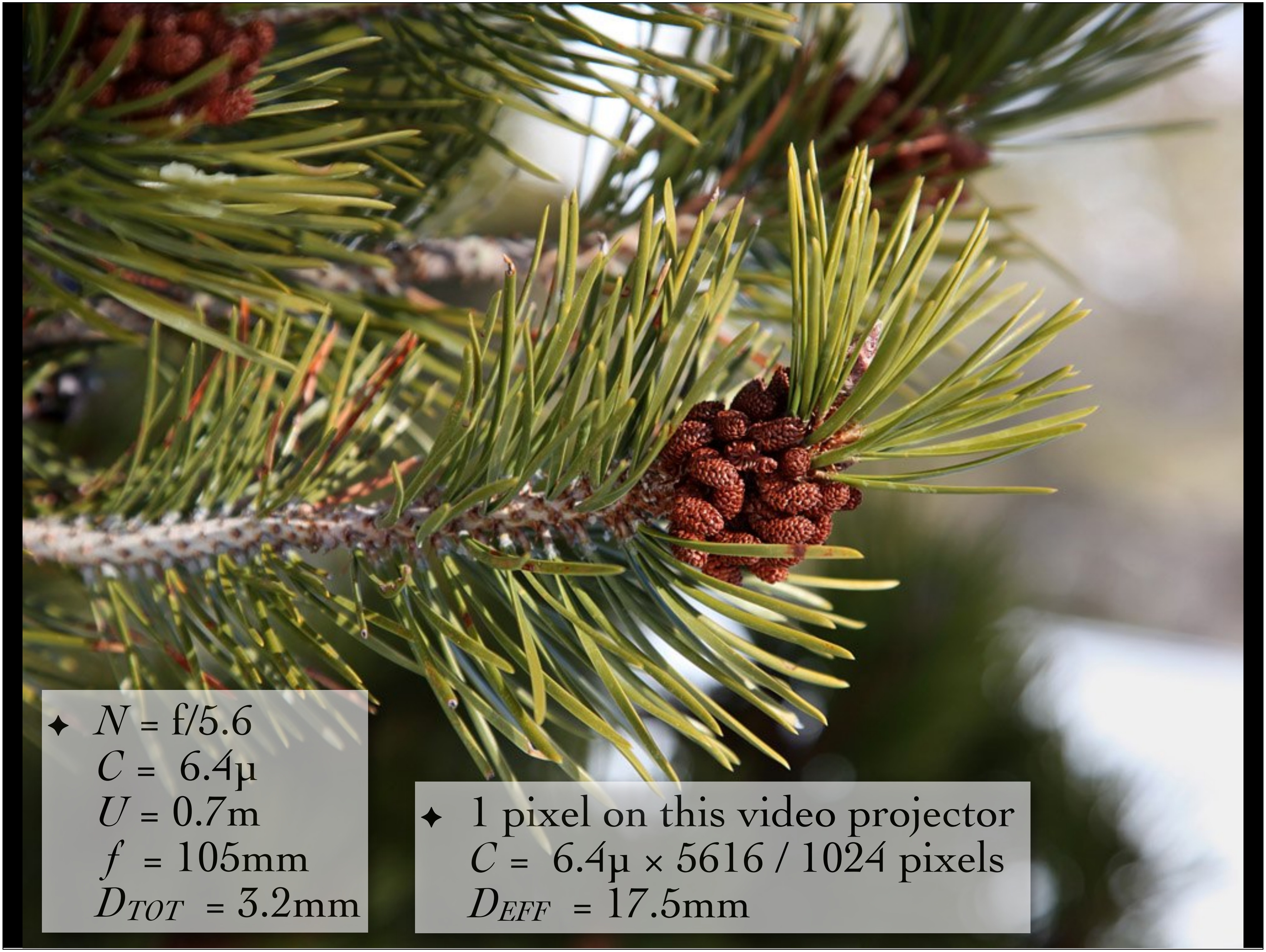
The "equiv to 362mm" is not used in the depth of field formula. I've provided it solely so that you see that this is a strongly telephoto shot. In other words, if I had used a 35mm full-frame camera, I would have used a 362mm lens to get this shot.

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

- N = f/4.1
  C = 2.5µ
  U = 5.9m (19')
  f = 73mm (equiv to 362mm)
  D_{TOT} = 132mm

- 1 pixel on this video projector
  C = 2.5µ × 2816 / 1024 pixels
  D_{EFF} = 363mm


✦ N = f/6.3
C = 2.5μ
U = 17m (56')
f = 27mm (equiv to 135mm)
D_{TOT} = 12.5m (41')
✦ 1 pixel on this video projector
C = 2.5μ × 2816 / 1024 pixels
D_{EFF} = 34m (113')



- N = f/5.6
  C = 6.4µ
  U = 0.7m
  f = 105mm
  D_{TOT} = 3.2mm

- 1 pixel on this video projector
  C = 6.4µ × 5616 / 1024 pixels
  D_{EFF} = 17.5mm

An alert student has pointed out that my original subject distance U of 31mm must be wrong, since it is less than focal length f, which is impossible. I grabbed this example from the Internet, and I now assume the photographer was quoting distance from the front lens element, which is not the same as U in a thin lens approximation.

Fortunately, there's an easy way to compute the correct subject distance. Using the Gaussian lens formula $1/s_o + 1/s_i = 1/f$, and knowing that $f = 65mm$ and $s_i/s_o = 5:1$, we can substitute and calculate that $s_o = 390mm$ and $s_i = U = 78mm$. This changes D_{TOT} to 0.29mm, which is still very small. I've fixed the slide. Note that since the lens isn't physically 390mm long, it must be a telephoto design!

Canon MP-E
65mm 5:1 macro

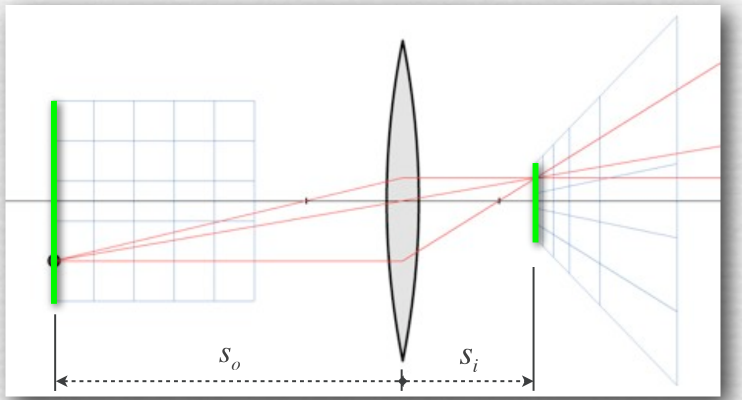
(Mikhail Shlemov)

- N = f/2.8
  C = 6.4µ
  U = 78mm
  f = 65mm
  (use $N' = (1+M_T)N$ at short conjugates (M_T=5 here)) = f/16
  D_{TOT} = 0.29mm!

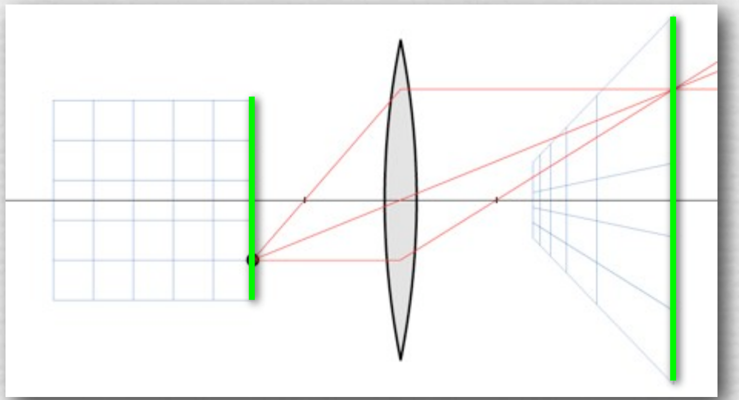
Sidelight: macro lenses

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Q. How can the Casio EX-F1 at 73mm and the Canon MP-E 65mm macro, which have similar *f*'s, have such different focusing distances?



normal



macro

- A. Because they are built to allow different s_i
  - this changes s_o, which changes magnification $M_T \triangleq -s_i / s_o$
  - macro lenses allow long s_i and they are well corrected for aberrations at short s_o

Extension tube: converts a normal lens to a macro lens

- toilet paper tube, black construction paper, masking tape
- camera hack by Katie Dektar (CS 178, 2009)

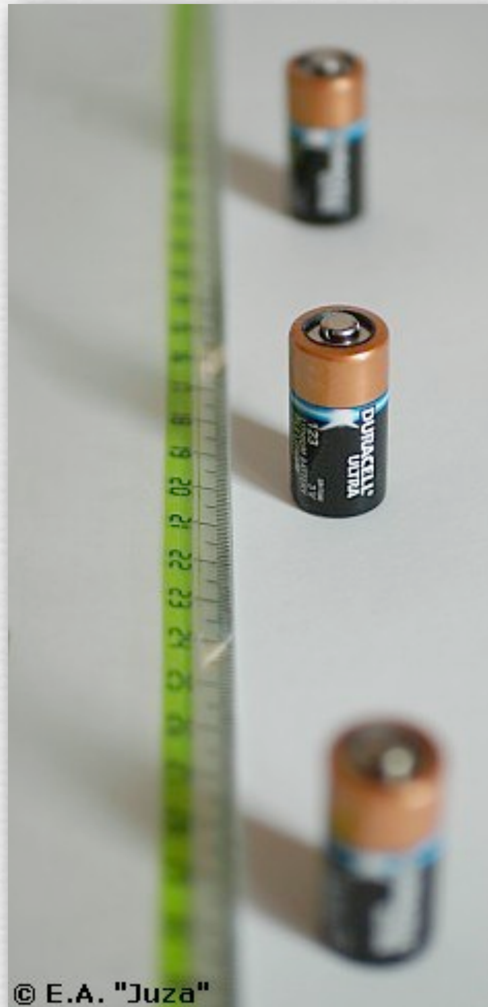
DoF is linear with F-number

$$D_{TOT} \approx \frac{2\boxed{N}CU^2}{f^2}$$

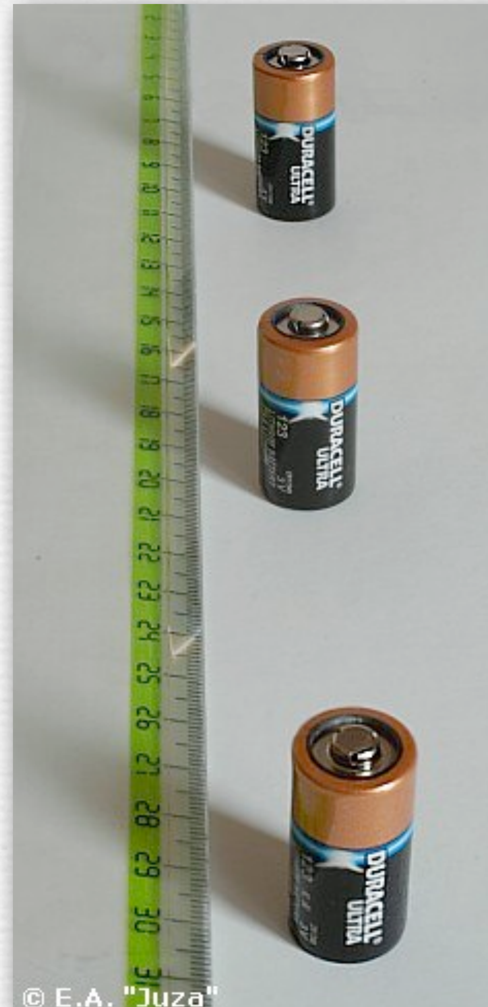
(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/dof.html

(juzaphoto.com)



f/2.8



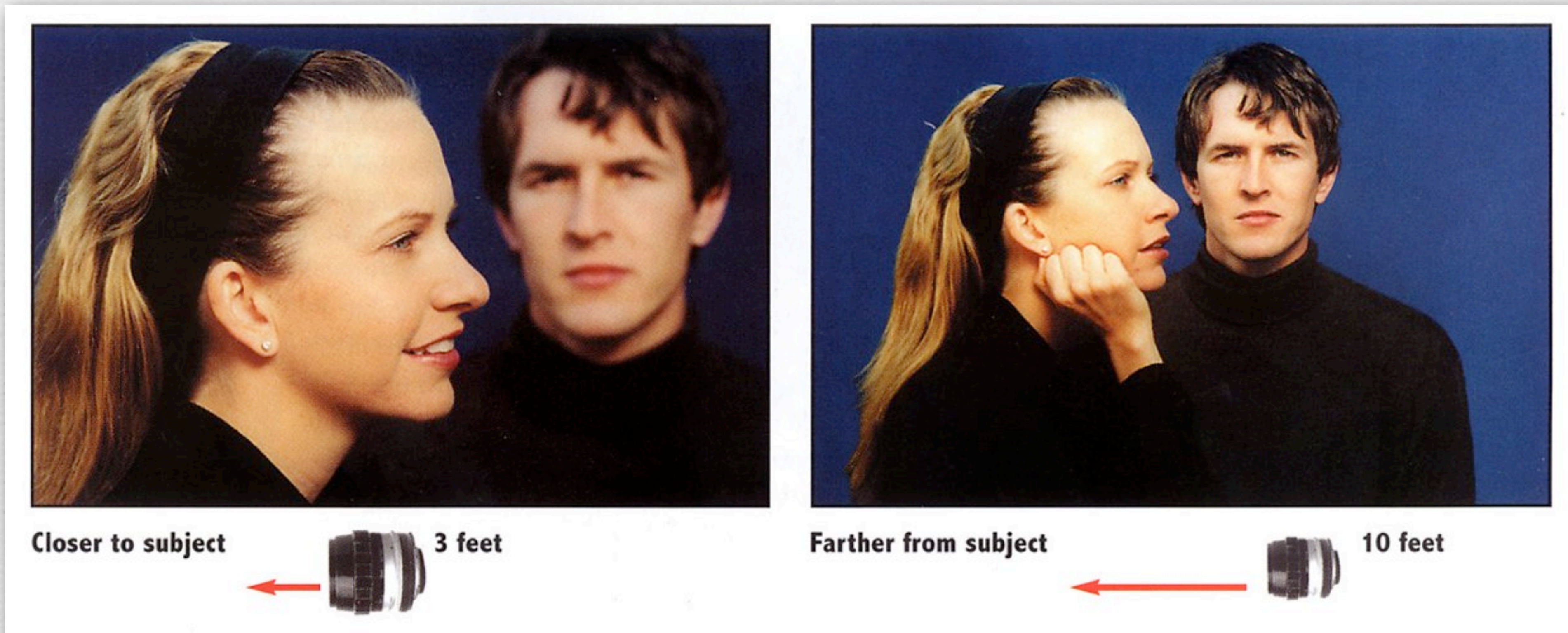
f/32

DoF is quadratic with subject distance

$$D_{TOT} \approx \frac{2NC\boxed{U^2}}{f^2}$$

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/dof.html



(London)

Hyperfocal distance

- the back depth of field

$$D_2 = \frac{NCU^2}{f^2 - NCU}$$

- becomes infinite if

$$U \geq \frac{f^2}{NC} \triangleq H$$

- In that case, the front depth of field becomes

$$D_1 = \frac{H}{2}$$

- so if I had focused at 32m, everything from 16m to infinity would be in focus on a video projector, including the men at 17m

- N = f/6.3
  C = 2.5μ × 2816 / 1024 pixels
  U = 17m (56')
  f = 27mm (equiv to 135mm)
  D_{TOT} = 34m on video projector
  H = 32m (106')

(FLASH DEMO)

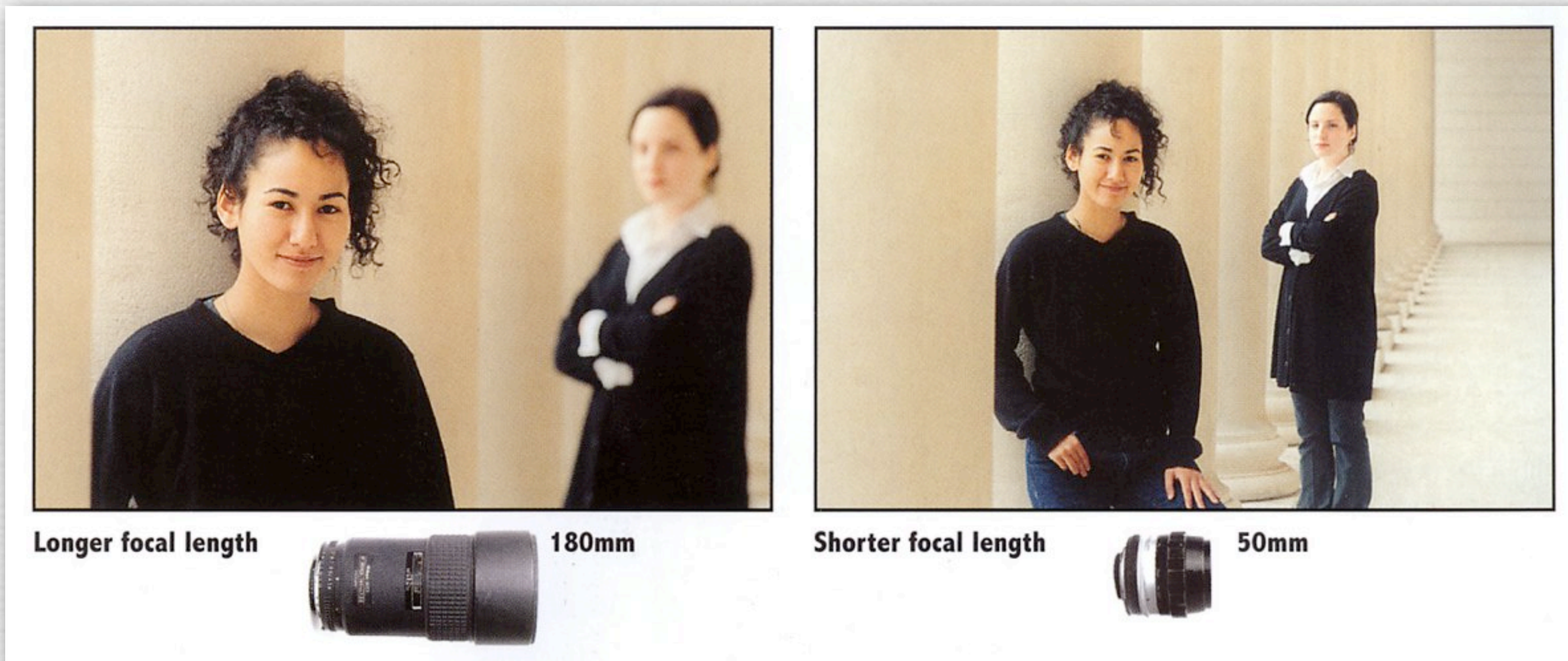
http://graphics.stanford.edu/courses/cs178/applets/dof.html

DoF is inverse quadratic with focal length

$$D_{TOT} \approx \frac{2NCU^2}{\boxed{f^2}}$$

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/dof.html



(London)

Q. Does sensor size affect DoF?

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

- as sensor shrinks, lens focal length f typically shrinks to maintain a comparable field of view
- as sensor shrinks, pixel size C typically shrinks to maintain a comparable number of pixels in the image
- thus, depth of field D_{TOT} increases linearly with decreasing sensor size
- this is why amateur cinematographers are drawn to SLRs
  - their chips are larger than even pro-level video camera chips
  - so they provide unprecedented control over depth of field

Vincent Laforet, Nocturne (2009)
Canon 1D Mark IV

DoF and the dolly-zoom

- if we zoom in (change f) and stand further back (change U) by the same factor

$$D_{TOT} \approx \frac{2NC\boxed{U^2}}{\boxed{f^2}}$$

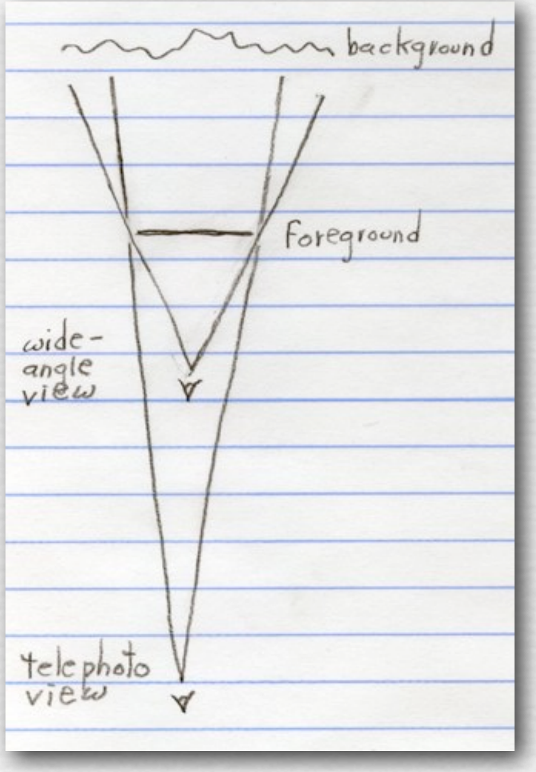
- the depth of field at the subject stays the same!
  - useful for macro when you can't get close enough

50mm f/4.8

(juzaphoto.com)

200mm f/4.8, moved back 4× from subject

Macro photography using a telephoto lens (contents of whiteboard)



- changing from a wide-angle lens to a telephoto lens and stepping back, you can make a foreground object appear the same size in both lenses
- and both lenses will have the same depth of field on that object
- but the telephoto sees a smaller part of the background (which it blows up to fill the field of view), so the background will appear blurrier

Parting thoughts on DoF: the zen of *bokeh*

(wikipedia.org)

Canon 85mm
prime f/1.8 lens

- the appearance of small out-of-focus features in a photograph with shallow depth of field
  - determined by the shape of the aperture
  - people get religious about it
  - but not every picture with shallow DoF has evident bokeh...

Natasha Gelfand (Canon 100mm f/2.8 prime macro lens)

Games with bokeh

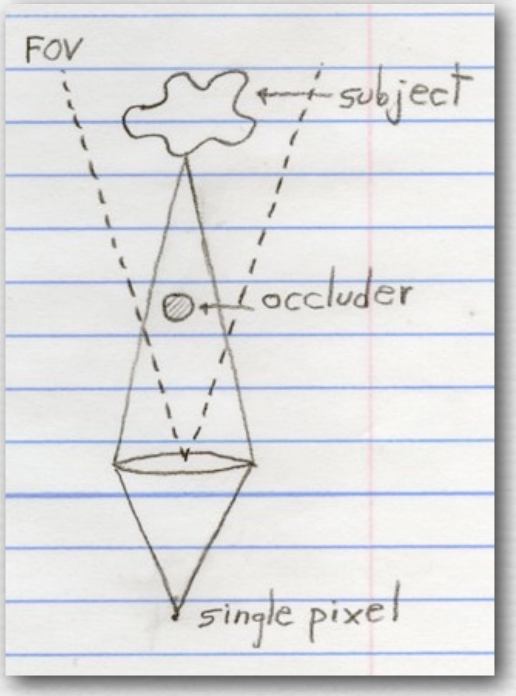
- picture by Alice Che (CS 178, 2010)
  - heart-shaped mask in front of lens
  - subject was Christmas lights
  - photograph was misfocused and under-exposed

Parting thoughts on DoF: seeing through occlusions

(Fredo Durand)

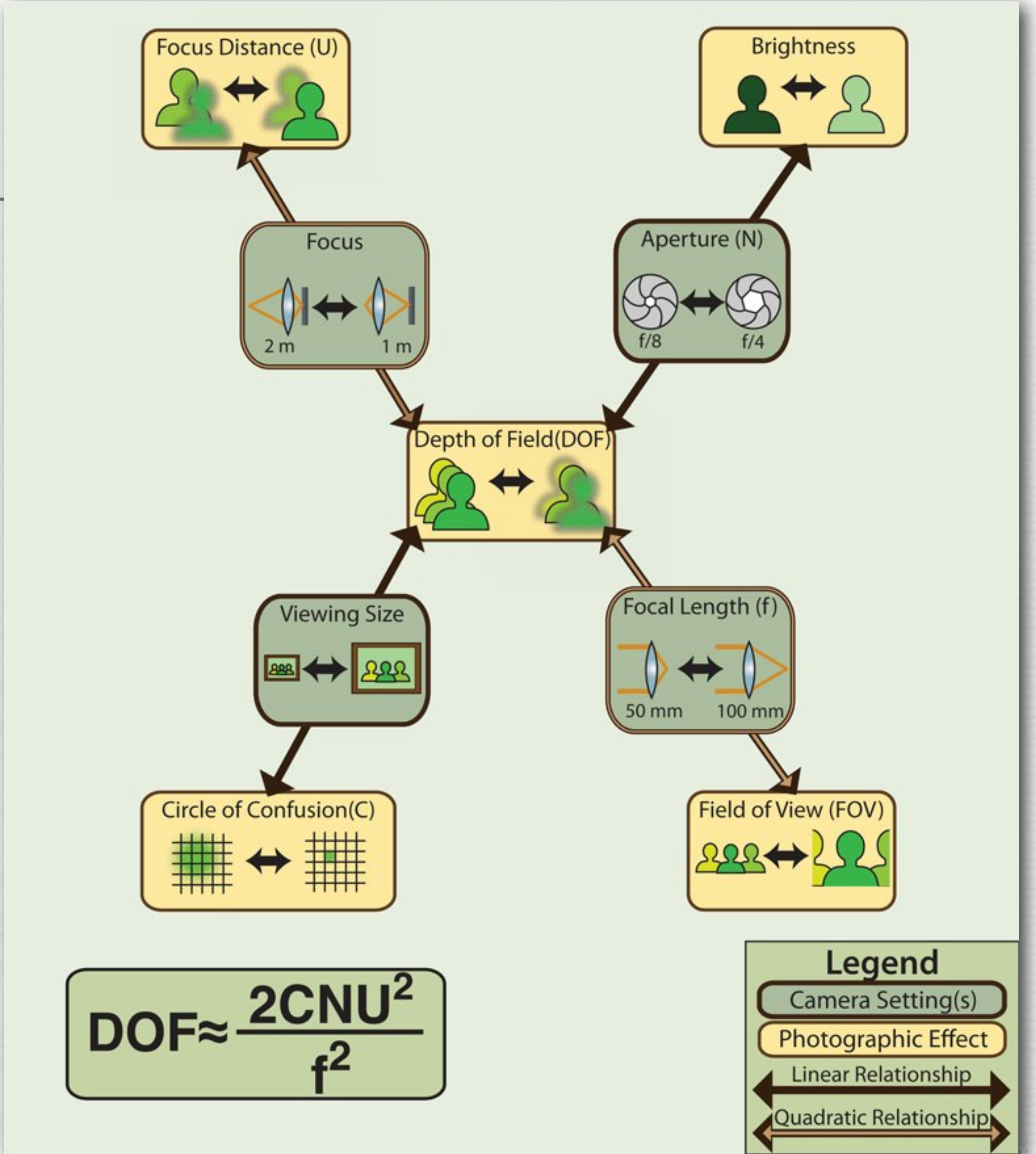
- depth of field is not a convolution of the image
  - i.e. not the same as blurring in Photoshop
  - DoF lets you eliminate occlusions, like a chain-link fence

Seeing through occlusions using a large aperture (contents of whiteboard)



- for a pixel focused on the subject, some of its rays will strike the occluder, but some will pass to the side of it, if the occluder is small enough
- the pixel will then be a mixture of the colors of the subject and occluder
- thus, the occluder reduces the contrast of your image of the subject, but it doesn't actually block your view of it

Tradeoffs affecting depth of field



(Eddy Talvala)

Recap

- depth of field (D_{TOT}) is governed by circle of confusion (C), aperture size (N), subject distance (U), and focal length (f)

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

  - depth of field is linear in some terms and quadratic in others
  - if you focus at the hyperfocal distance $H = f^2 / NC$, everything from $H / 2$ to infinity will be in focus
  - depth of field increases linearly with decreasing sensor size

- useful sidelights
  - bokeh refers to the appearance of small out-of-focus features
  - you can take macro photographs using a telephoto lens
  - depth of field blur is not the same as blurring an image

Questions?