# Light and reflection

CS 178, Spring 2010

Updated 5/19/10 and recap slides added.

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# Outline

- measures of light
  - radiometry versus photometry
  - radiant intensity of a point light
  - radiance leaving an area light
  - radiance arriving on a surface
  - irradiance on a surface
- reflection of light
  - diffuse
  - specular
  - goniometric diagrams
  - Fresnel equations and other effects

# Radiometry versus photometry

- *radiometry* is the study of light w/o considering humans
  - spectroradiometer - power as a function of wavelength
  - radiometer - total power, integrating over all wavelengths
  - measurements include
    - radiant intensity, radiance, irradiance

- *photometry* is the study of light as seen by humans
  - spectrophotometer - power we see as a function of wavelength
  - photometer, a.k.a. photographic light meter
  - measurements include
    - luminous intensity, luminance, illuminance

# Relationship to tristimulus theory

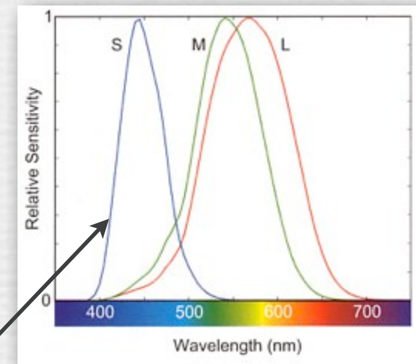
- the response of the human visual system to a spectrum is

$$(\rho,\gamma,\beta) \;=\; \left( \int_{400\,nm}^{700\,nm} L_e(\lambda)\,\rho(\lambda)\,d\lambda,\; \int_{400\,nm}^{700\,nm} L_e(\lambda)\,\gamma(\lambda)\,d\lambda,\; \int_{400\,nm}^{700\,nm} L_e(\lambda)\,\beta(\lambda)\,d\lambda \right)$$

radiance

luminance

(Stone)



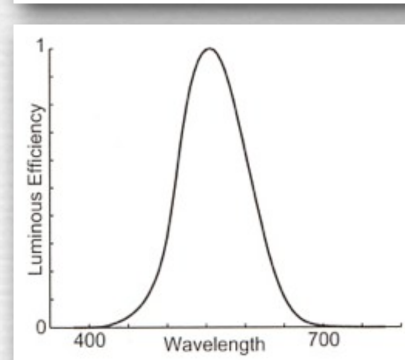

- the total response can be expressed as

$$L \;=\; \rho + \gamma + \beta \;=\; \int_{400\,nm}^{700\,nm} L_e(\lambda)\,V(\lambda)\,d\lambda$$

- where

$$V(\lambda) \;=\; \rho(\lambda) + \gamma(\lambda) + \beta(\lambda)$$

S is actually much lower than M or L




- one could also express luminance as

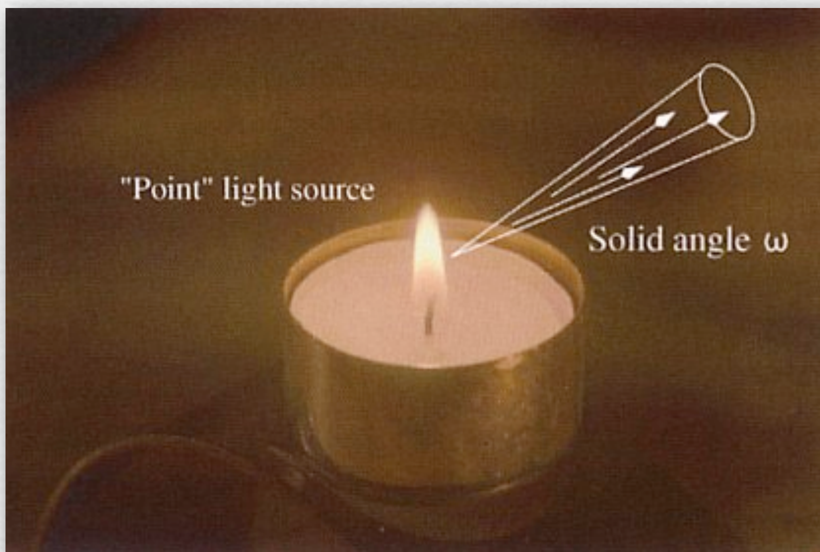
$$L(\lambda) \;=\; L_e(\lambda)\,V(\lambda)$$

*V(λ)*, or luminous efficiency curve

# Radiant intensity of a point light

- power given off by the light per unit solid angle

$$I = \frac{P}{\Omega} \qquad \left( \frac{\text{watts}}{\text{steradian}} \right)$$



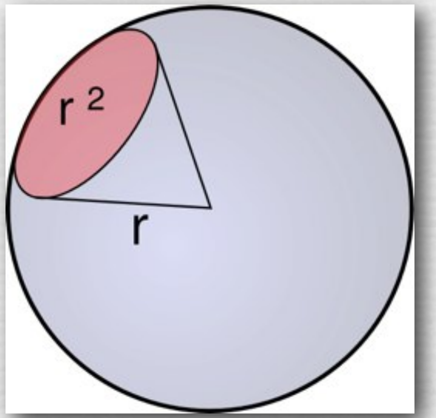

(Reinhard)

- i.e. the energy per unit time per unit solid angle
  - 1 watt = 1 joule / second

# Steradian as a measure of solid angle

- 1 steradian (sr) is the solid angle such that the area subtended by that solid angle on the surface of a sphere is equal to the sphere's radius$^2$
  - area of a sphere is $4\pi r^2$, so 1 sr = $r^2 / 4\pi r^2$ ≈ 1/12 of the sphere's surface

(http://www.handprint.com/HP/WCL/color3.html)




- examples
  - circular aperture 65° in subtended diameter
  - square aperture 57° on a side
  - a circle 12.7' in diameter cast by a streetlight 10' high

# Radiant intensity of a point light

- power given off by the light per unit solid angle

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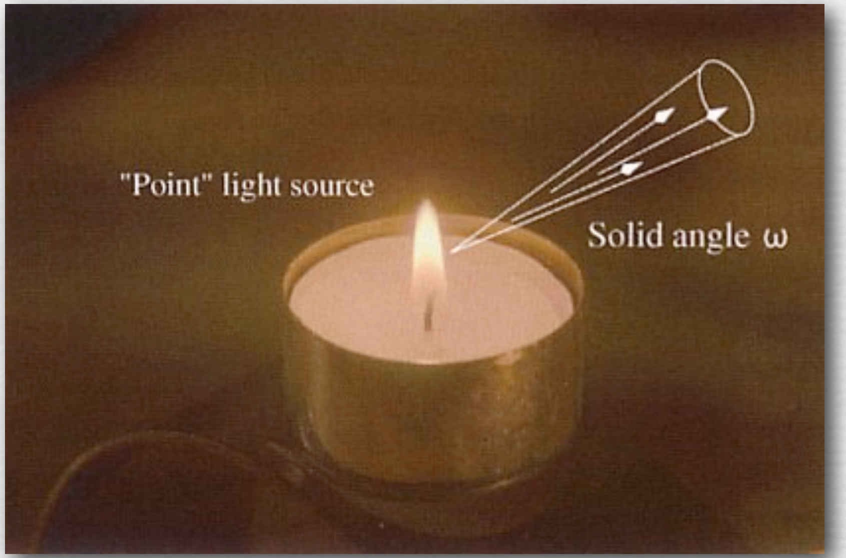
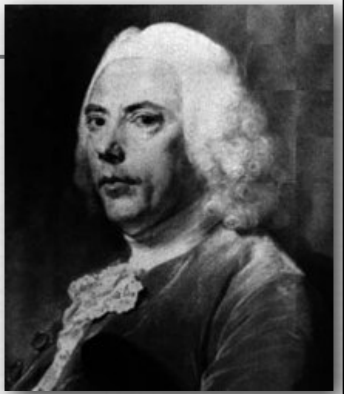



(Reinhard)

(http://www.handprint.com/HP/WCL/color3.html)

- example
  - 100W light bulb gives off 100 watts over the sphere ÷ $4\pi$ sr in a sphere = 8 watts within a 12.7' circle 10' feet from the bulb

# Radiant intensity of a point light



Pierre Bouguer
(1698-1758)

- power given off by the light per unit solid angle

$$I = \frac{P}{\Omega} \qquad \left( \frac{\text{watts}}{\text{steradian}} \right)$$

- related photometric concept is luminous intensity (measured in candelas)
  - 1 candela = 1 lumen / sr

If the light bulb were 100% efficient (i.e. no energy wasted as heat outside the visible spectrum), it would give off 683 lumens per watt. The ratio between 100 watts and 683 lumens represents the luminous efficiency of the human visual system across the visual spectrum.

- examples
  - a standard Bouguer candle gives off 1 candela
  - a 100W light bulb with a luminous efficiency of 2.6% (the other 97.4% we don't see) gives off 17.6 lumens per watt × 100W ÷ 4π sr in the sphere = 140 candelas

# Photography by candlelight

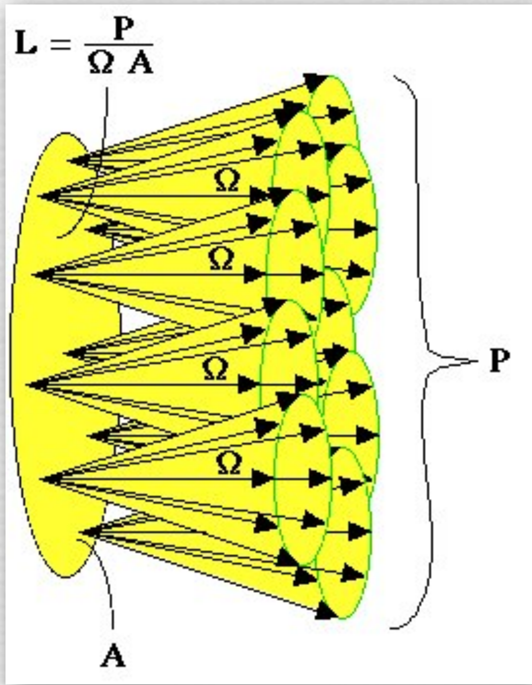
(digital-photography-school.com)

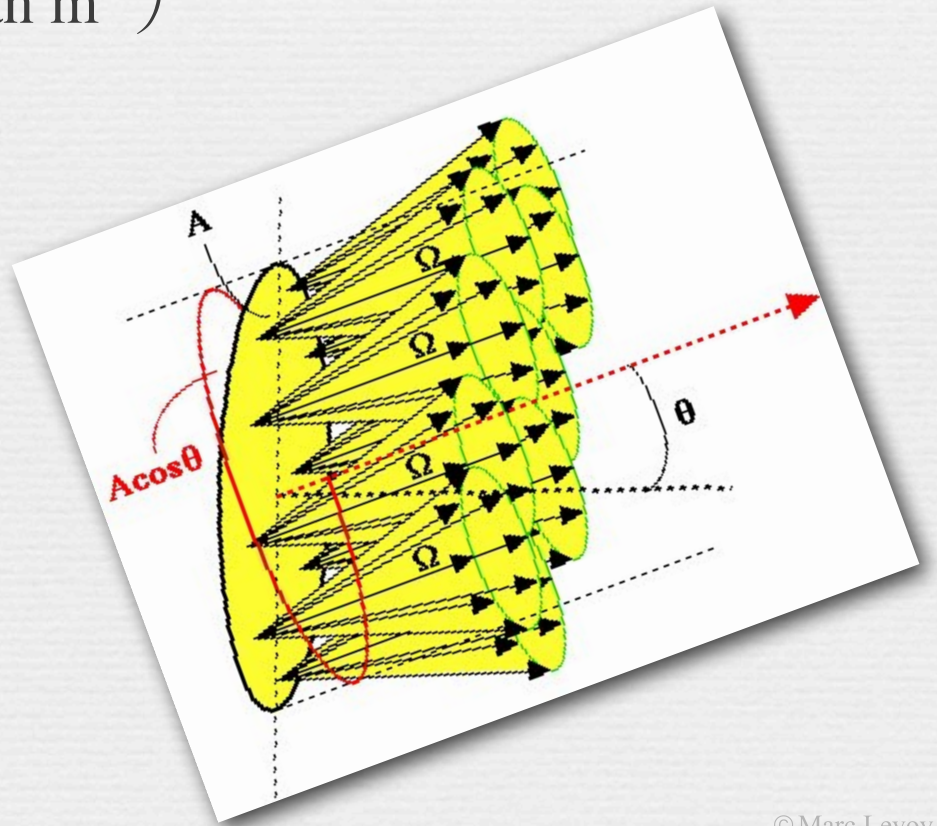
- need SLR-sized pixels, fast lens, tripod, patient subject
  - moderate shutter speed (1/15 sec) and ISO (400)

# Radiance leaving an area light

- power given off by the light per unit solid angle per unit area, viewed at a declination of θ relative to straight-on

$$L = \frac{P}{\Omega A \cos\theta} \qquad \left( \frac{\text{watts}}{\text{steradian m}^2} \right)$$




(http://omlc.ogi.edu/classroom/ece532/class1/radiance.html)

# Radiance leaving an area light

- power given off by the light per unit solid angle per unit area, viewed at a declination of θ relative to straight-on

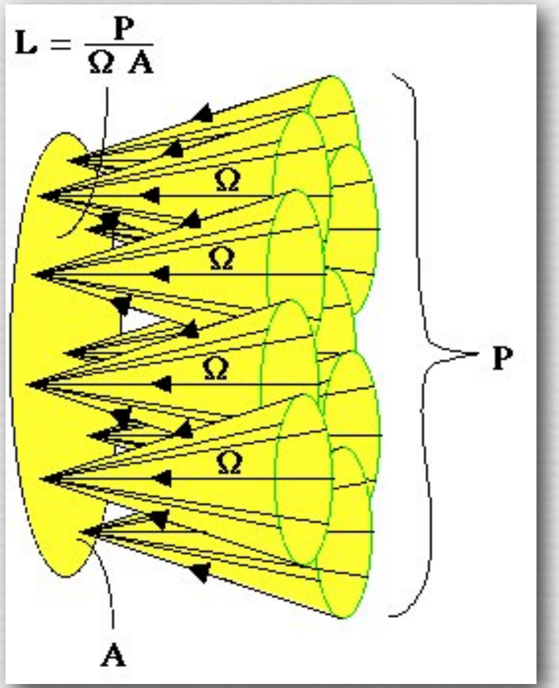
$$L = \frac{P}{\Omega A \cos\theta} \qquad \left(\frac{\text{watts}}{\text{steradian m}^2}\right)$$

- related photometric concept is luminance (measured in nits) (yup, nits!)
  - 1 nit = 1 candela / $m^2$ = 1 lumen / sr $m^2$
- example
  - viewed perpendicularly, a computer display gives off 50-300 candelas per meter$^2$ of the display surface, about the same as a 100W light bulb but spread out

# Radiance arriving on a surface

- power arriving on a surface per unit solid angle per unit area, illuminated from a declination of θ

$$L = \frac{P}{\Omega A \cos\theta} \qquad \left(\frac{\text{watts}}{\text{steradian m}^2}\right)$$



# Radiance arriving on a surface

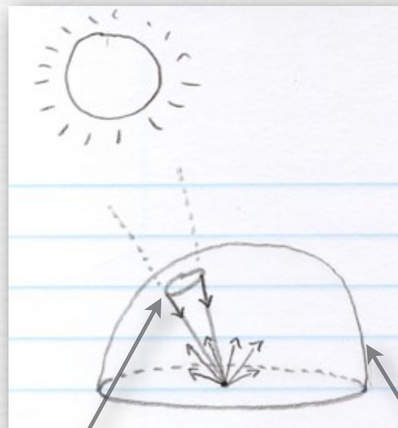
- power arriving on a surface per unit solid angle per unit area, illuminated from a declination of θ

$$L = \frac{P}{\Omega A \cos\theta} \qquad \left(\frac{\text{watts}}{\text{steradian m}^2}\right)$$

There is a proof missing here that the luminance leaving each point on the surface of the moon (per unit solid angle per unit area on the moon, at least for points that perpendicularly face the earth) is the same as the luminance arriving on each point of the earth (per unit solid angle per unit area on the earth, at least for points that perpendicularly face the moon). If you're interested in this proof that luminance (or radiance) is preserved during the transport of light, take CS 348B!

- examples (most are from Minnaert)
  - luminance arriving on a surface from a full (overhead) sun is 160,000 candelas/cm$^2$ (160,000 lumens/sr cm$^2$)
  - luminance reflected by a diffuse white surface illuminated by the sun is 1.6 cd/cm$^2$
  - reflected by a black surface is 0.04 cd/cm$^2$
  - arriving from a full overhead moon is 0.3 cd/cm$^2$
  - luminance arriving from a white cloud (fully lit by the sun) is 10 × luminance of the blue sky, a difference of 3.3 f/stops

# Luminance from sun → reflection from surface (contents of whiteboard)



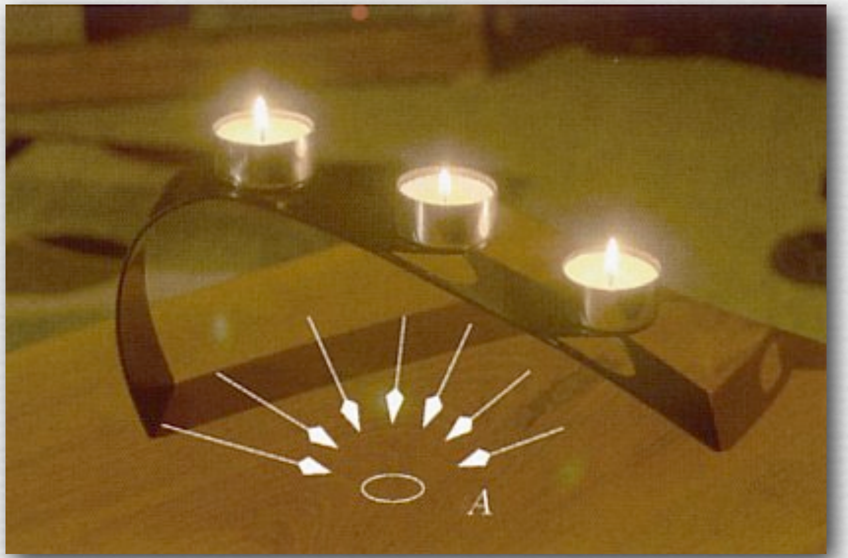
I may have muffed this derivation in class. Here it is worked out more completely, and with slightly more accurate numbers.

- Q. Why is the sun 160,000 candelas/$cm^2$ but its reflection by a diffuse white surface is only 1.6 cd/$cm^2$ ?
- A. the sun doesn't occupy the entire sky, but diffuse reflection does.
- luminance arrives from the sun through 0.001% of the celestial hemisphere (0.00006 sr), hence the amount arriving is 160,000 cd/$cm^2$ = lumens/sr $cm^2$ × 0.00006 sr = 10 lumens/$cm^2$
- if we assume a diffuse white surface reflects all the light it receives, then it reflects these 10 lumens/$cm^2$ into 100% of hemisphere ($2\pi$ sr), hence the surface's outgoing luminance is 10 lumens/$cm^2$ ÷ $2\pi$ sr = 1.6 lumens/sr $cm^2$ or 1.6 cd/$cm^2$

# Irradiance on a surface

- power accumulating on a surface per unit area, considering light arriving from all directions

$$E = \frac{P}{A} \qquad \left(\frac{\text{watts}}{\text{m}^2}\right)$$




(Reinhard)

# Irradiance on

I definitely muffed this derivation in class. if a Bouguer candle delivers 1 footcandle to a book surface if held 1 foot away, and 1 footcandle = 10.764 lux, then a candle delivers 10.764/0.25 = 43x as much irradiance as the full moon. To simulate the moon, and remembering that irradiance from a point source drops as the square of the distance between the source and receiving surface, I need to move the candle sqrt(43)x as far away = 6.6x away, or 6.6 feet away. This makes more sense than the 2 feet I derived in class, because I can't quite read by moonlight, but I can definitely read by the light from a candle 2 feet away. I'm going home tonight to try reading by a candle held 6.6 feet away!

- power accumulating on a surface per unit area, considering light arriving from all directions

$$E = \frac{P}{A} \qquad \left(\frac{\text{watts}}{\text{m}^2}\right)$$

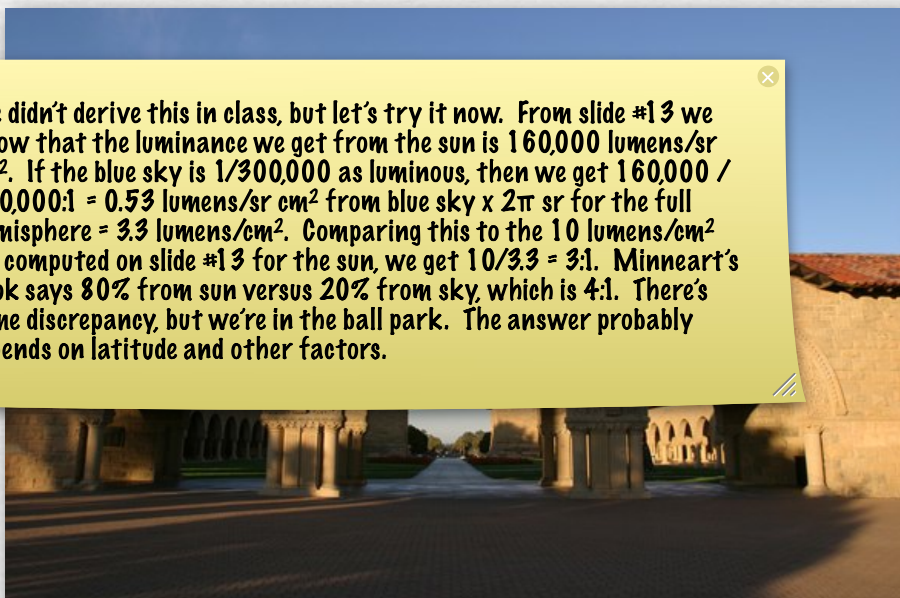
Q. How far from a book should I hold a candle to make it match the illumination of the moon?

- related photometric unit is illuminance (measured in lux)
  - 1 lux = 1 lumen / $m^2$
  - British unit is footcandle = 1 candela held 1 foot from surface (1 footcandle = 10.764 lux)
- example
  - illuminance from a bright star = illuminance from a candle 900 meters away = 1/810,000 lux
  - illuminance from the full moon = 1/4 lux

# How dark are outdoor shadows?

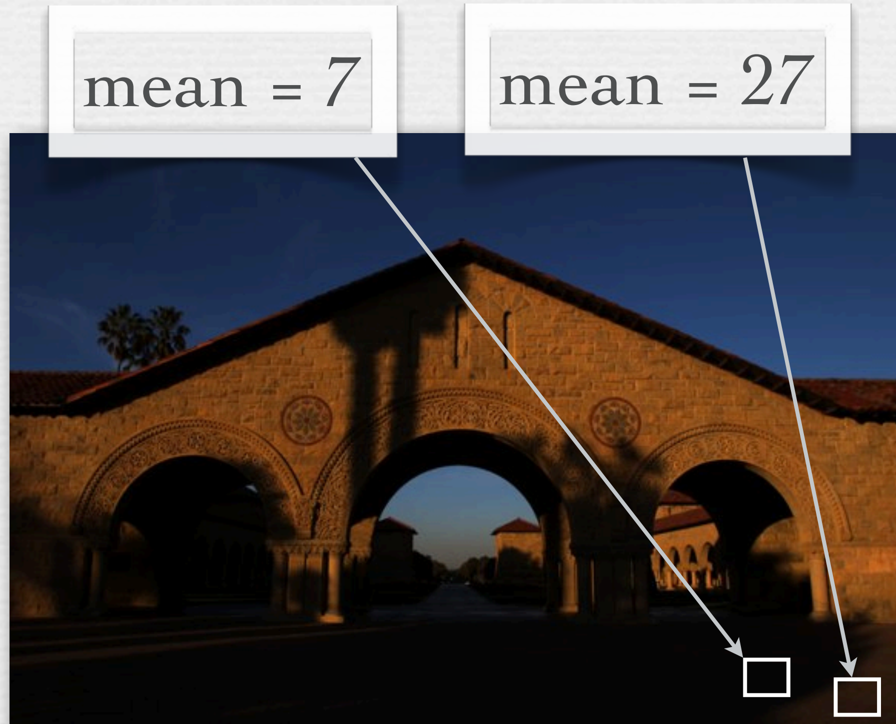
- luminance arriving on a surface from a full (overhead) sun is 300,000 × luminance arriving from the blue sky (from Minneart), but the sun occupies only a small fraction of the sky
- illuminance on a sunny day = 80% from the sun + 20% from blue sky, so shadows are 1/5 as bright as lit areas (2.3 f/stops)

(Marc Levoy)



We didn't derive this in class, but let's try it now. From slide #13 we know that the luminance we get from the sun is 160,000 lumens/sr cm$^2$. If the blue sky is 1/300,000 as luminous, then we get 160,000 / 300,000:1 = 0.53 lumens/sr cm$^2$ from blue sky x $2\pi$ sr for the full hemisphere = 3.3 lumens/cm$^2$. Comparing this to the 10 lumens/cm$^2$ we computed on slide #13 for the sun, we get 10/3.3 = 3:1. Minneart's book says 80% from sun versus 20% from sky, which is 4:1. There's some discrepancy, but we're in the ball park. The answer probably depends on latitude and other factors.

JPEG file




RAW, linearly boosted

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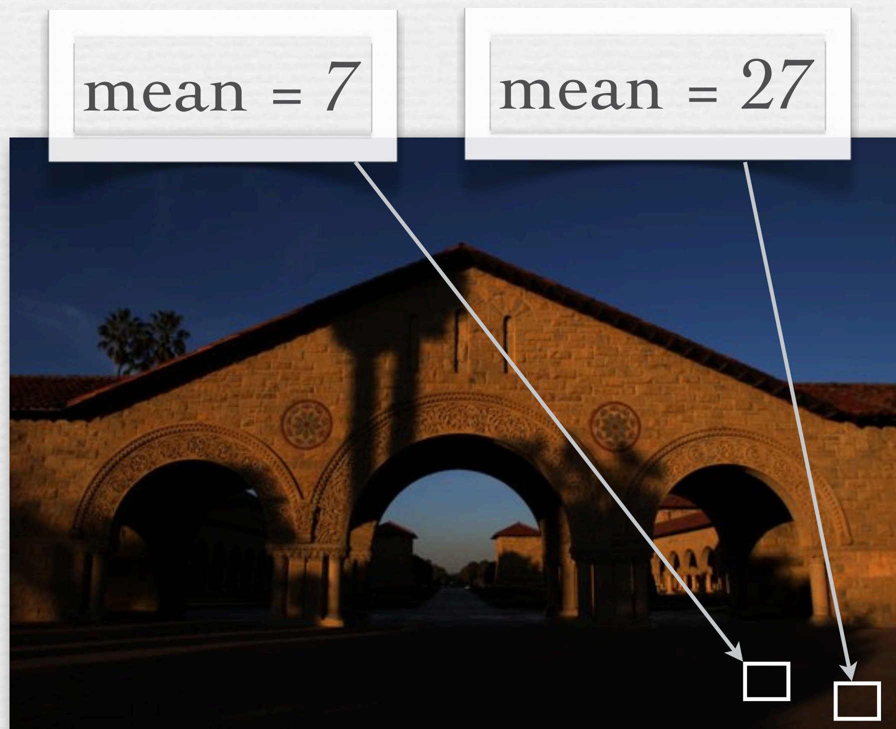



RAW, linearly boosted

# Recap

- to convert *radiometric* measures of light into *photometric* measures, multiply the spectral power distribution as measured by a spectroradiometer wavelength-by-wavelength by the human *luminous efficiency curve* V(λ)
- useful measures of light are the *radiant* or *luminous intensity* emitted by a point source (power per solid angle), the *radiance* or *luminance* emitted by (or arriving at) an area source (power per solid angle per unit area), and the *irradiance* or *illuminance* accumulating on a surface (power per unit area)
- bright objects (like the sun) may be more luminous (measured in lumens/sr $cm^2$) than darker objects (like the blue sky), but typically cover a smaller fraction of the incoming hemisphere
- outdoor shadows are 1/5 as bright as lit areas (2.3 f/stops)

Questions?

# Outline

- measures of light
  - radiometry versus photometry
  - radiant intensity of a point light
  - radiance leaving an area light
  - radiance arriving on a surface
  - irradiance on a surface
- ☞ reflection of light
  - diffuse
  - specular
  - goniometric diagrams
  - Fresnel equations and other effects

# Reflection from diffuse surfaces

(Dorsey)

two viewpoints, same illumination

Johann Lambert
(1728-1777)

- rough surfaces reflect light uniformly in all directions
  - appearance is independent of viewing direction
  - if perfectly so, surface is called ideal diffuse ("Lambertian")

# Albedo

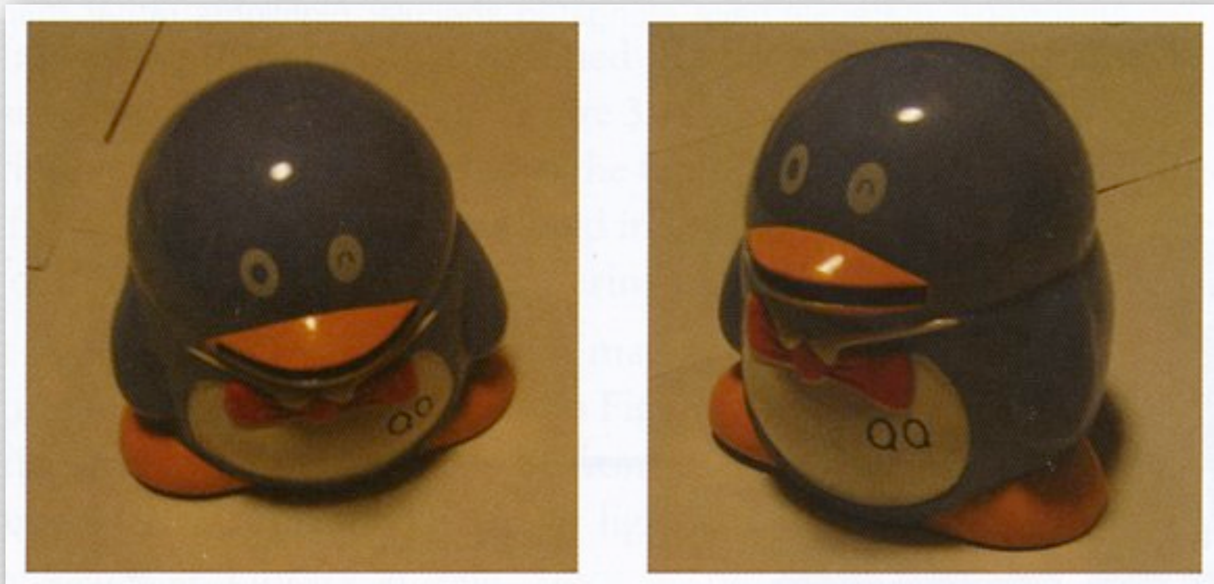
- fraction of light reflected from a diffuse surface
  - usually refers to an average across the visible spectrum

- examples
  - clouds 80%
  - fresh snow 80%
  - old snow 40%
  - grass 30%
  - soil 15%
  - rivers 7%
  - ocean 3%

equality explains "whiteout" in blizzards

not including mirror reflections of the sun

# Reflection from shiny surfaces

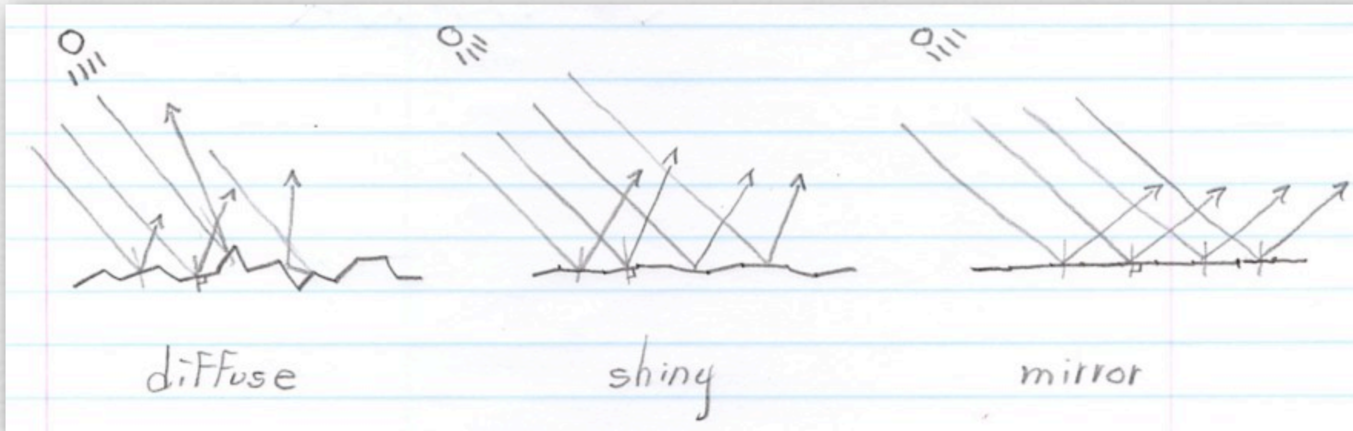



(Dorsey)

two viewpoints, same illumination (i.e. fixed to object)

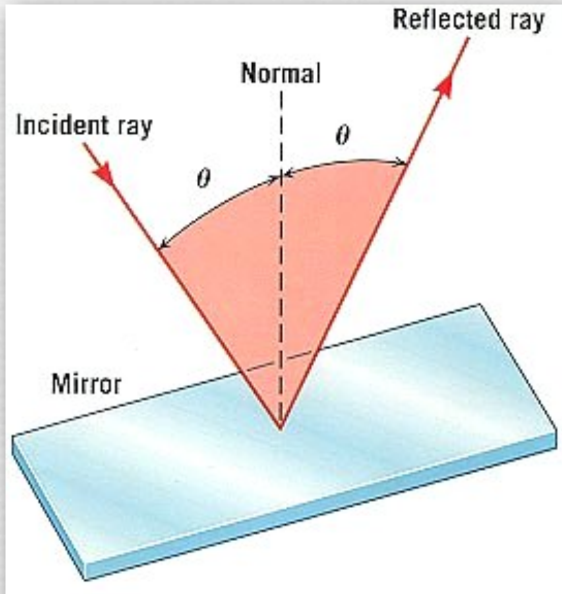
- rough surfaces are composed of flat microfacets ("asperities" according to Bouguer)
  - the amount of variance in the orientation of the facets determines whether the surface is *diffuse* or *specular*
  - diffuse reflections look the same regardless of viewing direction
  - specular reflections move when the light or observer moves

# Microfacet distributions (contents of whiteboard)

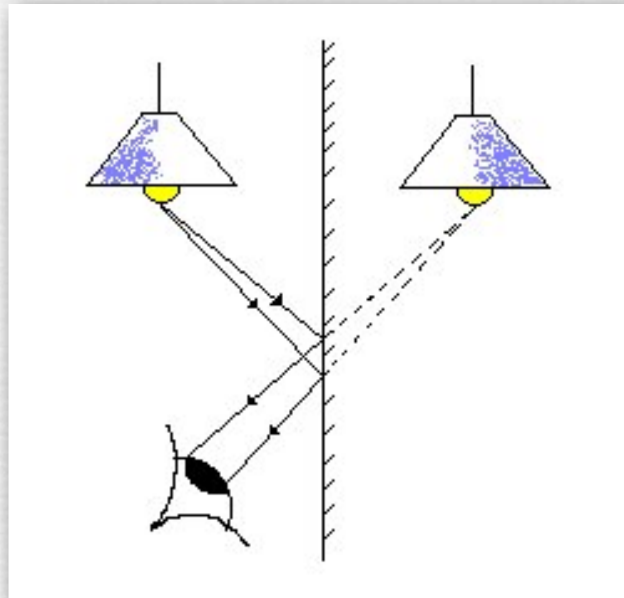



- if the facets are randomly oriented, and the variation in their orientation is large, then the surface appears *diffuse*
- if most of the facets are aligned with the surface, then it appears *specular* (a.k.a. shiny), with its *specular highlight* centered around the mirror reflection direction (angle of reflection = angle of incidence)
- if the surface is polished until no facets exist, then it is a mirror, and the angle of reflection = angle of incidence

# Mirror reflections



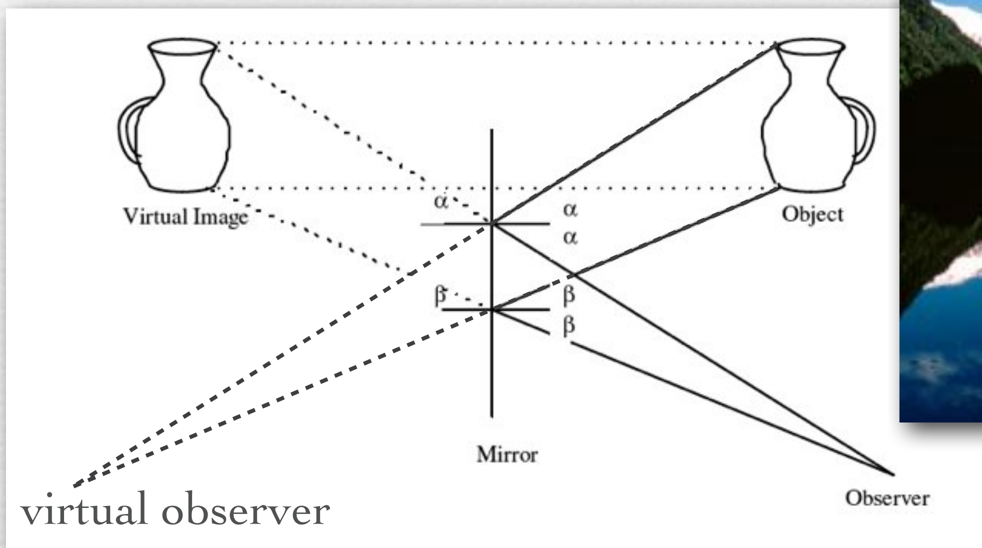

(Joseph Alward)



(ectc.org.uk)

- the focus distance of objects seen in mirrors is more than the distance from you to the mirror!

# Mirror reflections





- scenes reflected in water are not copies of the scenes!
  - the reflection shows the underside of the bridge

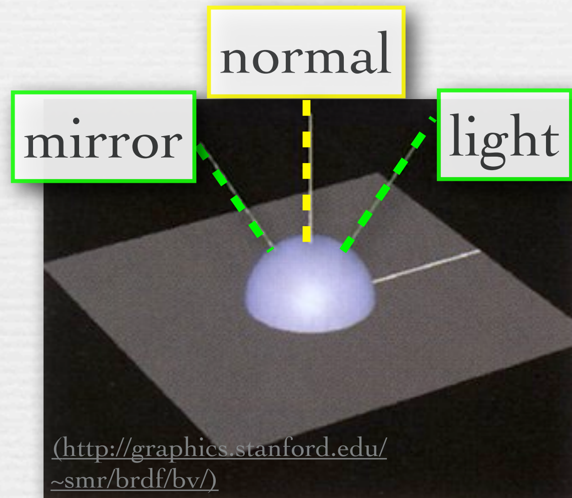
Diego Velázquez, Venus at her Mirror, 1647

Q. Who is Venus looking at in the mirror?

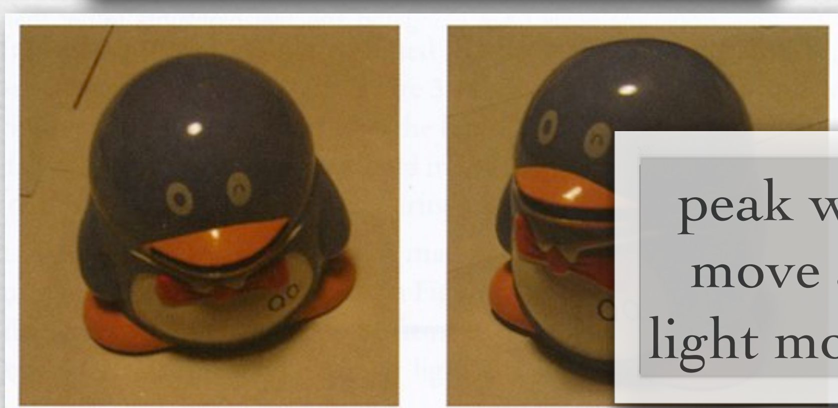
# Goniometric diagram

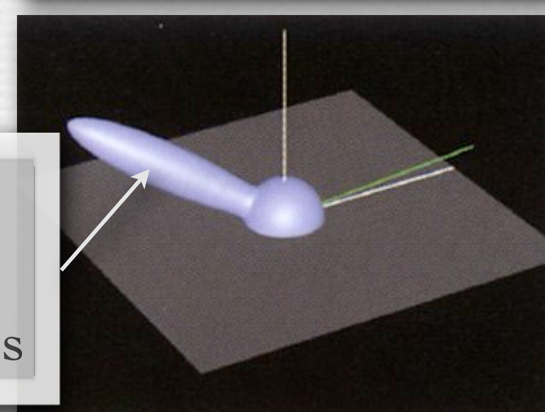
- depiction of reflectance (fraction of light reflected) as a function of one of the relevant angles or directions
- shown here is reflectance as a function of viewing direction, for a fixed incoming direction of light

diffuse surface



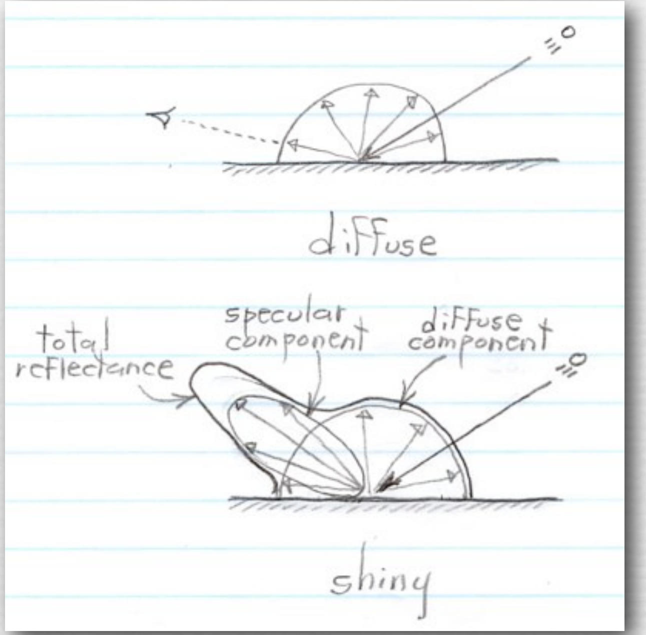

shiny surface





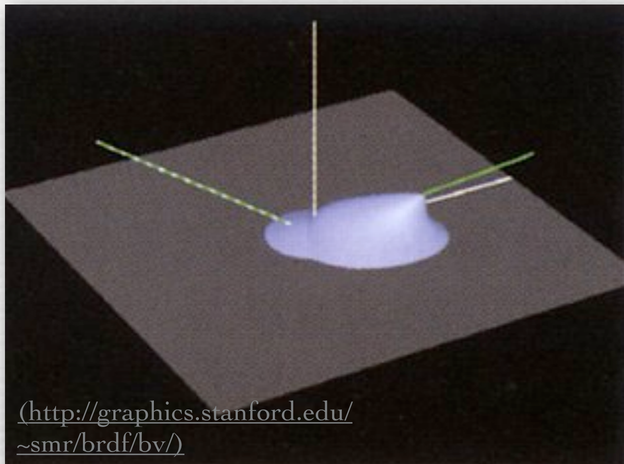
peak will move as light moves

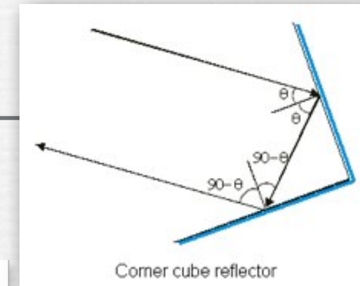
# Goniometric diagrams in flatland (contents of whiteboard)




- the incoming light is the long black vector at right in both drawings
- for the given incoming light direction, the fraction of light reflected in each viewing direction is given by the lengths of the small arrows
- in the shiny case, there is a *diffuse component*, whose reflectance is equal across all viewing directions, and a *specular component*, which is strongest in the mirror direction; the *total reflectance*, hence the final goniometric diagram, is the sum of these two components, i.e. the thick outer envelope

# What unusual material property does this goniometric diagram depict?

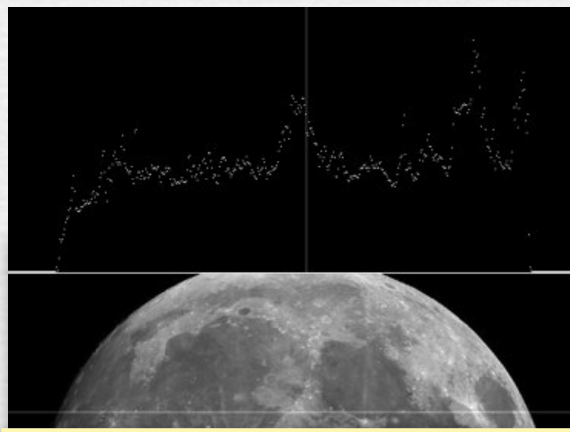



a full moon is roughly lit from the camera's viewpoint

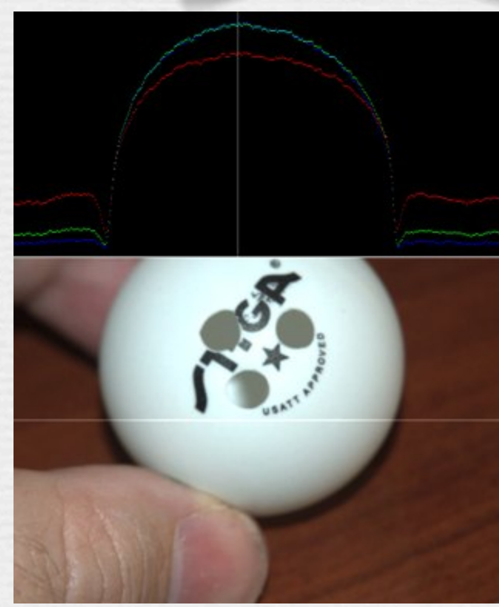
so is a flash photograph

- A. retroreflectivity
- the maria of the moon is retroreflective and gray
- a diffuse sphere, lit from the camera's viewpoint, falls off as cos θ or $\sqrt{1-x^2}$

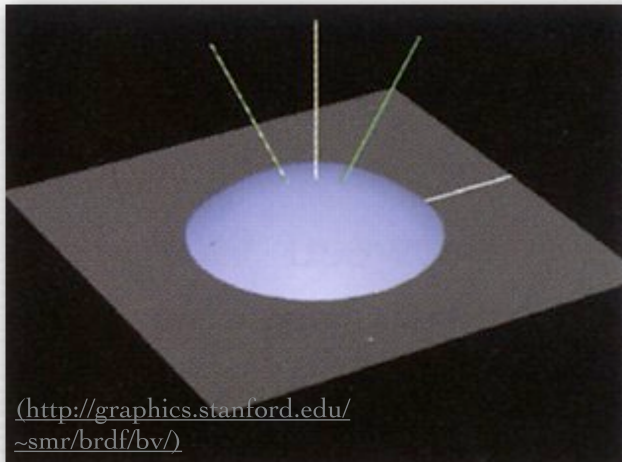


(NASA)

I didn't explain this in class, but irradiance drops as cos(θ), where θ is the angle between the illumination and the normal to the receiving surface, i.e. irradiance $E(\theta) = L_i \cos(\theta)$ where $L_i$ is the incident radiance from the flash, and $\theta(x) = \mathrm{sqrt}(1-x^2)$, if x goes from 0 to 1 from the cnter to the edge of the ping pong ball. Then reflected radiance $L_r = k\, L_i$ regardless of θ for a Lambertian surface. If you want to learn more about doing these sorts of calculations, take CS 148!

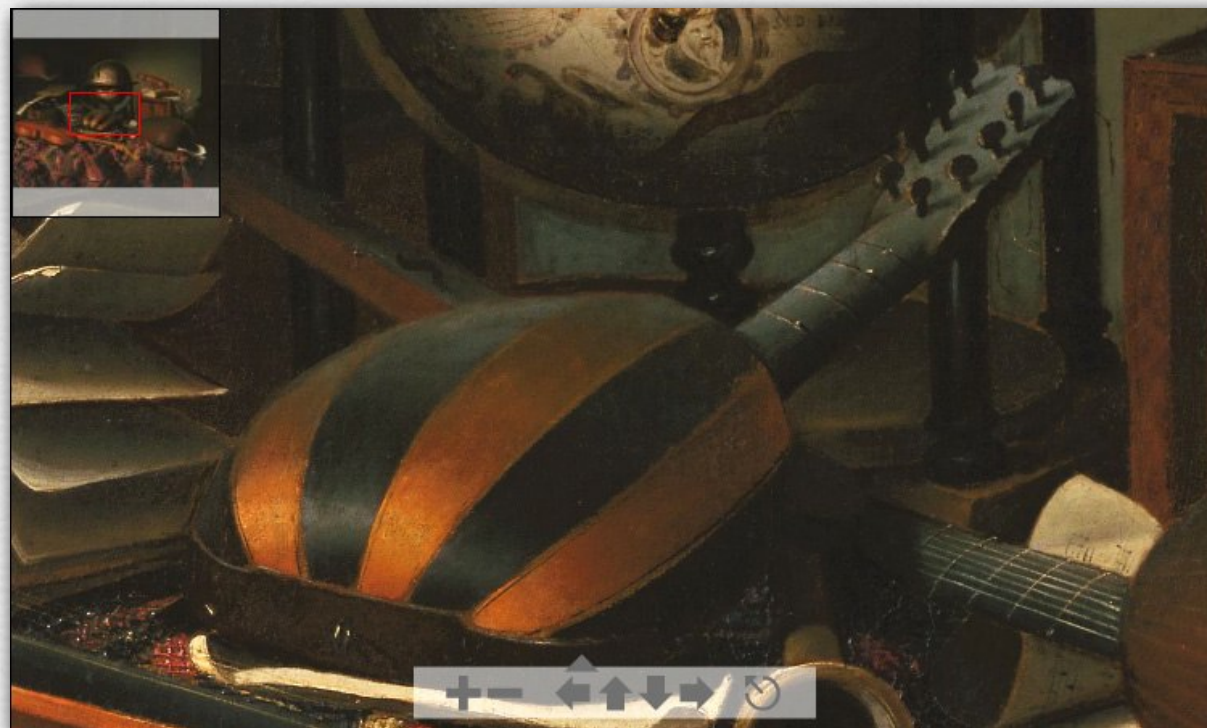


# What about this goniometric diagram?




- A. dusty scatterer
- appears brighter as the viewer moves to grazing angles

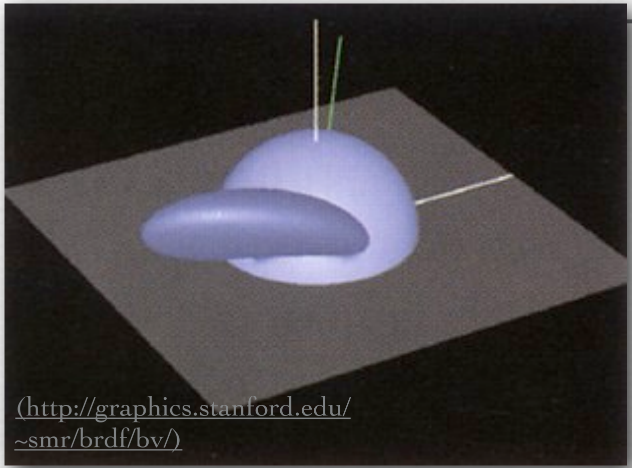
(famsf.org)

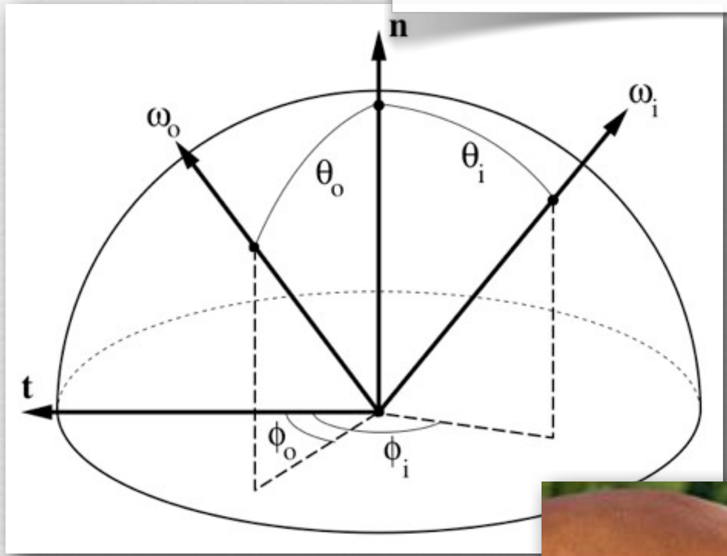


Bartolomeo Bettera,
Still Life with Musical Instruments,
17th century

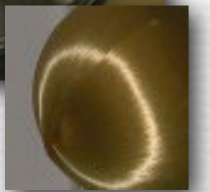
# And this goniometric diagram?

θ's denote declination;
ϕ's denote azimuth



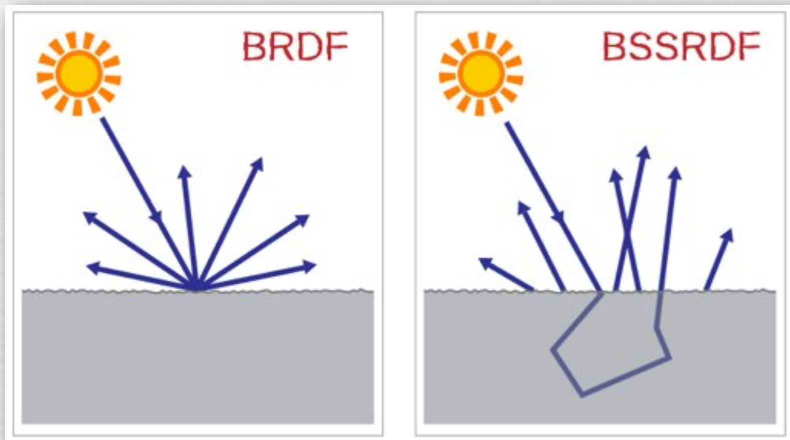

- A. anisotropic reflection
  - highlight not radially symmetric around mirror direction
  - highlight may depend on light direction $\phi_i$ and viewer direction $\phi_o$ (like the horse), or only on the difference $\phi_i$ - $\phi_o$ between them (pot and Xmas tree ornament)
- produced by grooved or directionally textured materials

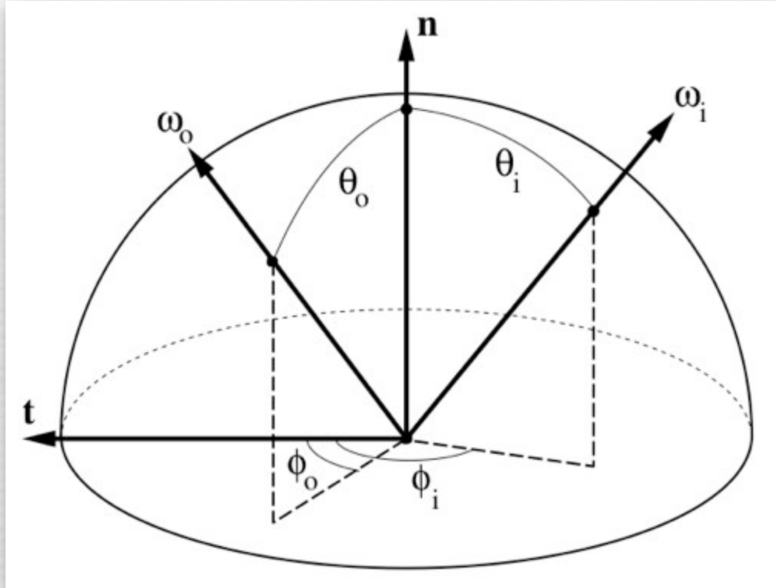


(horsemanmagazine.com)

# BRDFs and BSSRDFs

(wikipedia)




(http://graphics.stanford.edu/~smr/brdf/bv/)

- Bidirectional Reflectance Distribution Function (BRDF, 4D function)

$$f_r(\theta_i, \phi_i, \theta_r, \phi_r) \qquad \left(\frac{1}{sr}\right)$$

- Bidirectional Surface Scattering Reflectance Distribution Function (BSSRDF, 8D function)

$$\rho(x_i, y_i, \theta_i, \phi_i, x_r, y_r, \theta_r, \phi_r) \qquad \left(\frac{1}{sr}\right)$$

# BRDFs versus BSSRDFs

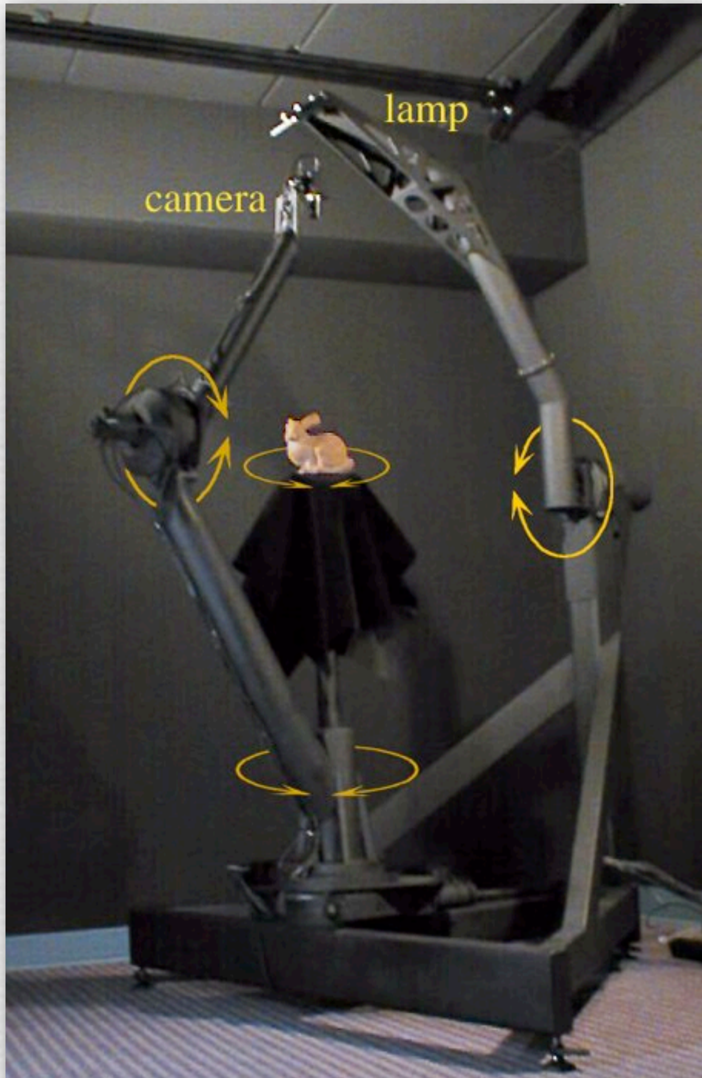
(Henrik Wann Jensen)

BRDF

BSSRDF

- subsurface scattering is critical to the appearance of human skin
  - cosmetics hide blemishes, but they also prevent subsurface scattering

# Devices for measuring BRDFs




Stanford Spherical Gantry

(Henrik Wann Jensen)

# Fresnel equations

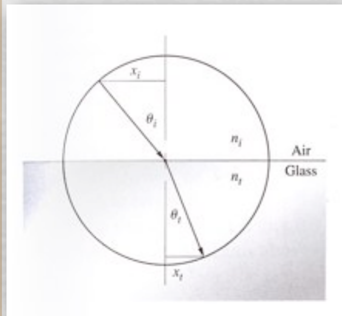
- a model of reflectance derived from physical optics (light as waves), not geometrical optics (light as rays)

Augustin-Jean Fresnel
(1788-1827)

(wikipedia)

$$R_s = \left[\frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}\right]^2 = \left(\frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}\right)^2 = \left[\frac{n_1 \cos\theta_i - n_2\sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta_i\right)^2}}{n_1 \cos\theta_i + n_2\sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta_i\right)^2}}\right]^2$$

$$R_p = \left[\frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)}\right]^2 = \left(\frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i}\right)^2 = \left[\frac{n_1\sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta_i\right)^2} - n_2 \cos\theta_i}{n_1\sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta_i\right)^2} + n_2 \cos\theta_i}\right]^2$$




(Hecht)

- effects
  - conductors (metals) - specular highlight is color of metal
  - non-conductors (dielectrics) - specular highlight is color of light
  - specular highlight becomes color of light at grazing angles
  - even diffuse surfaces become specular at grazing angles

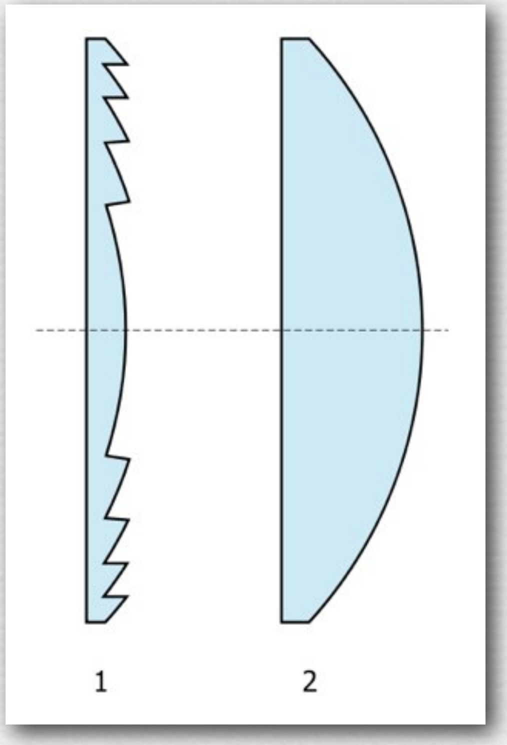
# Recap

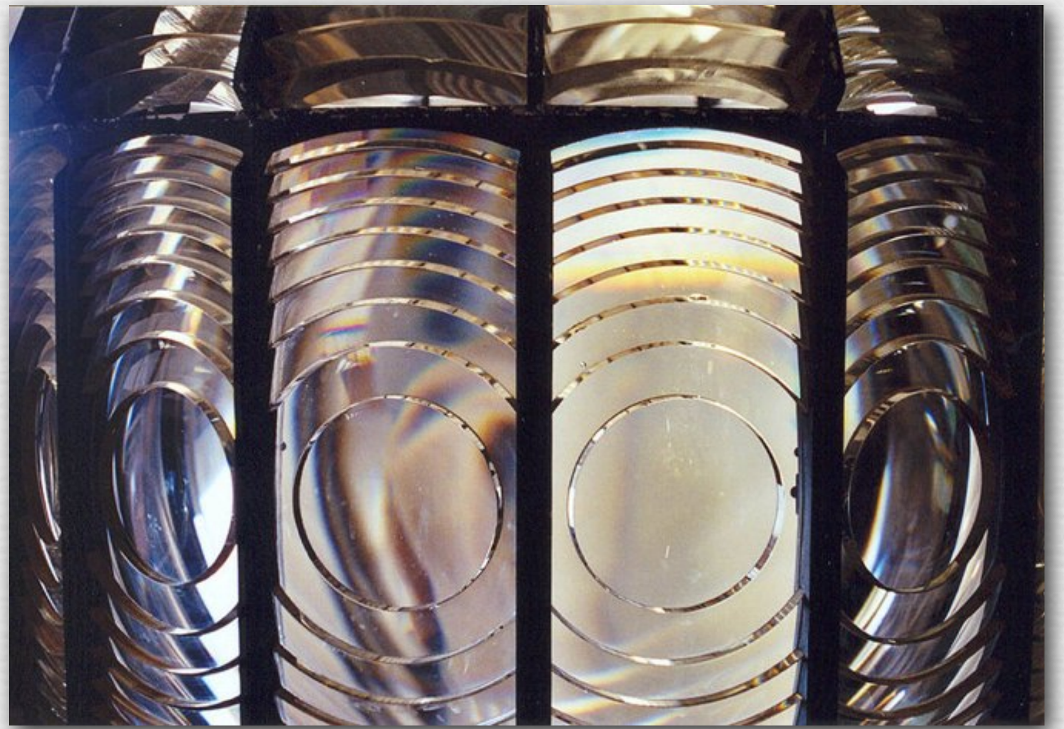
- rough surfaces (called *diffuse*) have *microfacets* of widely varying orientation, causing them to reflect light equally in all directions
- shiny surfaces (called *specular*) have microfacets with less variation in orientation, causing them to reflect light preferentially in the mirror direction, which changes with viewing direction
- goniometric diagrams give reflectance as a function of viewing direction for a given lighting direction; for a shiny surface, *total reflectance* for a viewing direction is the sum of a *diffuse component* and a *specular component*
- some materials are *retroreflective* or *scattering* or *anisotropic*
- the 4D *BRDF* characterizes reflectance as a function of incoming and outgoing direction; the 8D *BSSRDF* adds incoming and outgoing surface position, permitting characterization of *subsurface scattering*
- the *Fresnel equations* model additional effects, for example the reflectance from metals is the color of the metal, not the color of the light source

**Questions?**

# Fresnel Lens

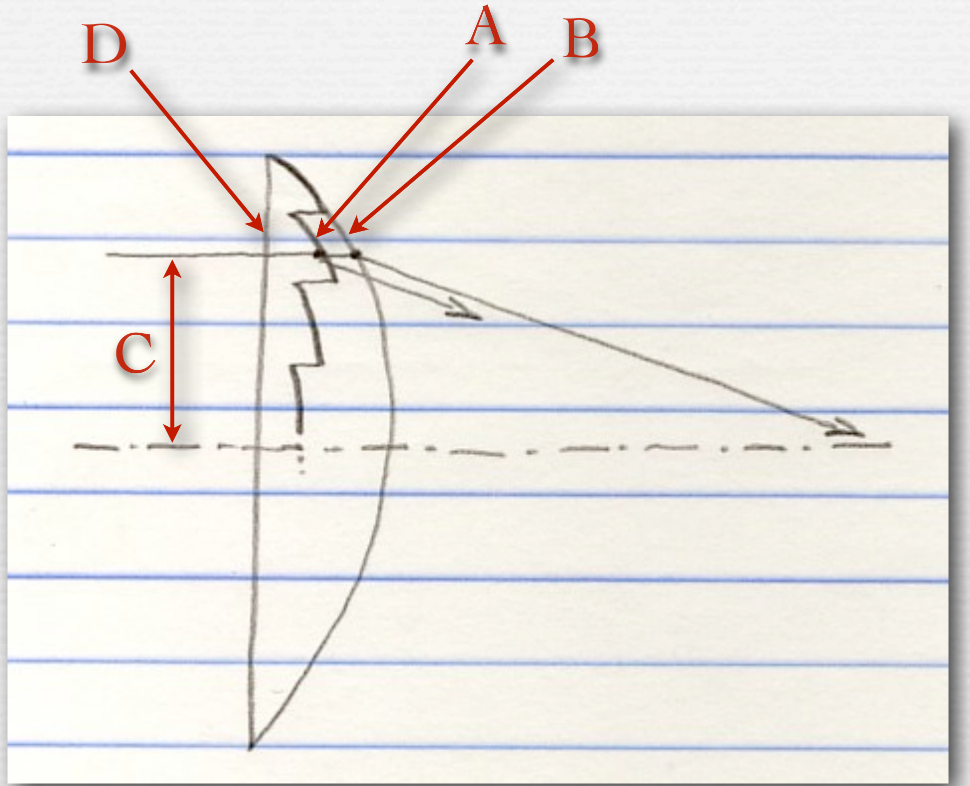
- same refractive power (focal length) as a much thicker lens
- good for focusing light, but not for making images



(wikipedia)

# The geometry of a Fresnel lens (contents of whiteboard)




- each Fresnel segment (A) is parallel to that part of the original lens (B) which is at the same ray height (distance from the optical axis (C)), but it's closer to the planar surface (D), making the lens physically thinner, hence lighter and cheaper

Tyler Westcott, Pigeon Point Lighthouse in light fog,
photographed during the annual relighting of its historical 1KW lantern, 2008
(Nikon D40, 30 seconds, ISO 200, not Photoshopped)

# Parting puzzle

- Q. These vials represent progressive stages of pounding chunks of green glass into a fine powder; why are they getting whiter?

(Gary Meyer)

# Slide credits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- Dorsey, J., Rushmeier, H., Sillion, F., *Digital Modeling of Material Appearance,* Elsevier, 2008.
- Reinhard et al., *High Dynamic Range Imaging*, Elsevier, 2006.
- Minnaert, M.G.J., *Light and Color in the Outdoors*, Springer-Verlag, 1993.